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Effect of heat transfer law on the finite-time exergoeconomic performance of a Carnot refrigerator

Lingen Chen [a], Chih Wu [b*], Fengrui Sun [a]

[a] *Faculty 306, Naval University of Engineering, Wuhan 430033, People's Republic of China*

[b] *Department of Mechanical Engineering, U.S. Naval Academy, Annapolis, MD 21402, USA*

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Abstract—The operation of a Carnot refrigerator is viewed as a production process with exergy as its output. The economic optimization of the endoreversible refrigerator is carried out in this paper. The Coefficient of Performance (COP) of the refrigerator is a secondary consideration of the practical engineering effort of maximizing cooling rate and exergy whose goodness is constrained by economical considerations. Therefore, the profit of the refrigerator is taken as the optimization objective. Using the method of finite-time exergoeconomic analysis, which emphasizes the compromise optimization between economics (profit) and the appropriate energy utilization factor (Coefficient of Performance, COP) for finite-time (endoreversible) thermodynamic cycles, this paper derives the relation between optimal profit and COP of an endoreversible Carnot refrigerator based on a relatively general heat transfer law $q \propto \Delta(T^n)$. The COP at the maximum profit is also obtained. The results obtained involve those for three common heat transfer laws: Newton's law ($n = 1$), the linear phenomenological law in irreversible thermodynamics ($n = -1$), and the radiative heat transfer law ($n = 4$). © 2001 Éditions scientifiques et médicales Elsevier SAS

Nomenclature

A	exergy output of refrigerator	kJ
C	cost	$\$\cdot s^{-1}$
D	function defined in equation (31)	$K\cdot kW^{-1}$
E	function defined in equation (29)	K
E_1	function defined in equation (68)	
F	function defined in equation (29)	K
F_1	function defined in equation (69)	
i	sequential variable	
n	sequential variable	
P	revenue per cycle	$\$\cdot s^{-1}$
P_{in}	power input	kW
q	specific heat transfer	kW
Q_1	heat flow from refrigerator to heat sink	kJ
Q_2	heat flow from heat source to cold refrigerant	kJ
R	cooling load	kW
R_m	cooling load at maximum profit	kW
R_{max}	maximum cooling load	kW
R_{min}	minimum cooling load	kW
t	time	s
T	temperature	K
T_0	environmental temperature	K
T_H	heat sink temperature	K
T_L	heat source temperature	K
T_{WH}	warm refrigerant temperature	K
T_{WL}	cold refrigerant temperature	K
W	work	kJ
Z_1	function defined in equation (10)	
Z_2	function defined in equation (11)	
Z_3	function defined in equation (12)	
Z_4	function defined in equation (17)	
Z_5	function defined in equation (49)	K
Z_6	function defined in equation (50)	K
Z_7	function defined in equation (51)	K
Z_8	function defined in equation (52)	K
Z_9	function defined in equation (60)	K
Z_{10}	function defined in equation (61)	K
Z_{11}	function defined in equation (62)	K
Z_{12}	function defined in equation (64)	kW
Z_{13}	function defined in equation (65)	kW
Z_{14}	function defined in equation (66)	kW

Greek letter

α	heat conductance	$kW\cdot K^{-1}$
β	heat conductance	$kW\cdot K^{-1}$

* Correspondence and reprints.
E-mail address: wu@gwmail.usna.edu (C. Wu).

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γ	temperature index defined in equation (33)	
γ_m	temperature index at the condition of maximum profit	
δ	function defined in equation (12)	
ε	COP	
ε_c	Carnot COP	
ε_m	COP at maximum profit, finite-time exergoeconomic COP bound	
ϵ_R	COP at maximum cooling load	
η	Carnot coefficient	
σ	rate of entropy production	$kW \cdot K^{-1}$
ΔS	change in entropy	$kJ \cdot K^{-1}$
τ	total time	s
ϕ_A	price of exergy	$\$ \cdot kJ^{-1}$
ϕ_W	price of work	$\$ \cdot kJ^{-1}$
ψ	profit ratio	$\$ \cdot s^{-1}$
Π_m	optimal profit	$\$ \cdot s^{-1}$
Π_{max}	maximum profit	$\$ \cdot s^{-1}$

1. INTRODUCTION

The Carnot engine proposed in 1824 operates on reversible process principles. As a consequence, this hypothetical engine produces the maximum possible work for a given heat source and sink temperatures, but generates zero power because it has to operate at an infinitely slow pace. Its thermodynamic efficiency, which has long been used as the standard against which all real engine efficiencies are measured, is unrealistically high. In 1975 did Curzon and Ahlborn [1] pioneered an analysis that accounts for the irreversibilities of finite-time heat transfer to and from the engine. Such an endoreversible engine can generate useful power. Because of external irreversibilities, its efficiency at maximum power, which is termed the "finite-time thermodynamic efficiency", is less than that of the Carnot efficiency. Since finite-time thermodynamics was first advanced in 1975, many authors have studied the effect of irreversibilities on the performance of thermodynamic processes and cycles. Some detailed literature surveys of finite-time thermodynamics were given by Sieniutycz and Salamon [2] and Chen et al. [3, 4]. Some authors [5–18, including] have assessed the effect of finite-rates of heat transfer on the performance of irreversible refrigerators.

The objective functions in finite-time thermodynamics are often pure thermodynamic parameters including power, efficiency, entropy production, effectiveness, cooling load, specific cooling load, COP and loss of exergy. Salamon and Nitzan [19] viewed the operation of the endoreversible heat engine as a production process with work as its output. They carried out the economic optimization of the heat engine with the maximum profit as the objective function [20].

A relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems is exergoeconomic (or thermoeconomic) analysis. Some detailed literature surveys of the exergoeconomics were given by Sieniutycz and Salamon [2] and Tsatsaronis [21]. Salamon and Nitzan's work [19] combined the endoreversible model with exergoeconomic analysis. We termed it as finite-time exergoeconomic analysis [22, 23] to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization. Similarly, we termed the performance bound at maximum profit as finite-time exergoeconomic performance bound to distinguish it from the finite-time thermodynamic performance bound at maximum thermodynamic output. Based on the work for heat engines [19, 22, 23], such a method has been extended to Newton's Law two-heat-reservoir refrigerator [10, 11] and heat pump [24], and the three-heat-reservoir refrigerator [25] and heat pump by Chen et al. [26].

Heat transfer affects the performance of endoreversible cycles. A few authors [7, 9, 13, including] have assessed the effect of heat transfer laws on the cooling load versus COP characteristics for a refrigerator. A new step taken in this paper is the estimation of the profit versus the COP characteristics and analysis of the finite-time exergoeconomic performance based on a relatively general heat transfer law, $q \propto \Delta(T^n)$, where n is a heat transfer exponent. The heat transfers obey Newton's Law when $n = 1$, the linear phenomenological law in irreversible thermodynamics when $n = -1$, and the radiative heat transfer law when $n = 4$.

2. ANALYSIS

2.1. The relation between optimal profit and cop

An endoreversible Carnot refrigerator is shown in *figure 1*. The only irreversible processes in the cycle are the two heat transfer processes from the refrigerator to the heat sink and from the heat source to the refrigerator. To analyze this cycle, we assume that the temperatures of the heat sink, heat source, warm refrigerant in the heat

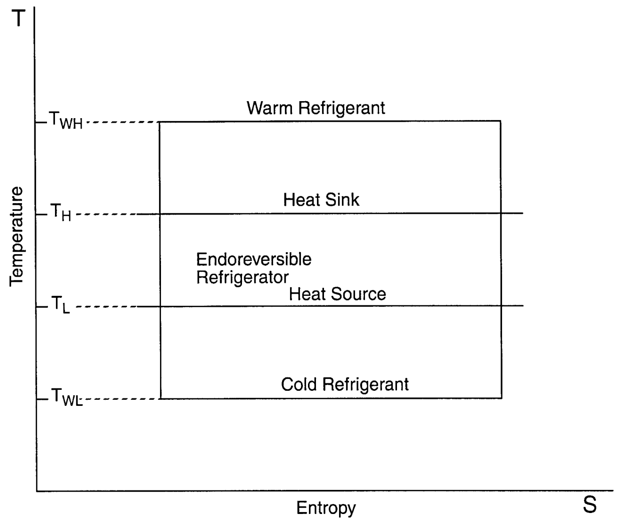

Figure 1. Endoreversible Carnot refrigerator.

rejection process, and cold refrigerant in the heat addition process are T_{H}, T_{L}, T_{WH}, and T_{WL}, respectively. Thus heat flows from the heat source to the cold refrigerant across a temperature difference of $(T_{\mathrm{L}} - T_{\mathrm{WL}})$ and heat flows from the warm refrigerant to the heat sink across a temperature difference of $(T_{\mathrm{WH}} - T_{\mathrm{H}})$. Assuming the heat transfers between the refrigerant and the reservoirs obey a generalized heat transfer law, $q \propto \Delta(T^n)$, then

$$Q_1 = \alpha\left(T_{\mathrm{WH}}^n - T_{\mathrm{H}}^n\right)t_1 \tag{1}$$

$$Q_2 = \beta\left(T_{\mathrm{L}}^n - T_{\mathrm{WL}}^n\right)t_2 \tag{2}$$

where Q_1 and Q_2 are heat flows from the refrigerator to the heat sink and from the heat source to the cold refrigerant. Constants α and β are the heat conductances (product of heat transfer coefficient and heat transfer surface area) between warm refrigerant and heat sink and between heat source and cold refrigerant. Variables t_1 and t_2 are the times required to transfer an amount Q_1 and Q_2 of heat, respectively.

Neglecting the time required for the two isentropic processes, the total time (τ) required for the whole cycle is:

$$\tau = t_1 + t_2 \tag{3}$$

The COP (ε) and the work input (W) to the refrigerator are

$$\varepsilon = (Q_1/Q_2 - 1)^{-1} = (T_{\mathrm{WH}}/T_{\mathrm{WL}} - 1)^{-1} \tag{4}$$

$$W = Q_1 - Q_2 \tag{5}$$

Assuming the environment temperature is T_0 and the following relation holds: $T_0 \geqslant T_{\mathrm{H}} > T_{\mathrm{L}}$, the exergy output of the refrigerator (A) is:

$$A = Q_2(T_0/T_{\mathrm{L}} - 1) - Q_1(T_0/T_{\mathrm{H}} - 1) = Q_2\eta_2 - Q_1\eta_1 \tag{6}$$

where η_i is the Carnot coefficient of the reservoir i.

The profit (Π) is calculated for the cycle period as follows. If ϕ_{A} is the value price of exergy output, we have a revenue function (P) per cycle:

$$P = \phi_{\mathrm{A}}A/\tau \tag{7}$$

We assume that the only input to the production process is the work input (W) taken from the motor. This corresponds to a cost per unit time (C):

$$C = \phi_{\mathrm{W}}W/\tau \tag{8}$$

where ϕ_{W} is the price of work.

Using equations (1)–(8), the profit of the refrigerator is obtained

$$\begin{aligned}\Pi = P - C &= \left[(Q_2\eta_2 - Q_1\eta_1)\phi_{\mathrm{A}} - (Q_1 - Q_2)\phi_{\mathrm{W}}\right]/\tau \\ &= \left[\alpha\phi_{\mathrm{A}}(Z_2 - Z_1Z_3)\right]/\left[Z_3\left(T_{\mathrm{WL}}^nZ_3^n - T_{\mathrm{H}}^n\right)^{-1} \right.\\ &\qquad \left. + \delta^2\left(T_{\mathrm{L}}^n - T_{\mathrm{WL}}^n\right)^{-1}\right]\end{aligned} \tag{9}$$

where

$$Z_1 = \eta_1 + \phi_{\mathrm{W}}/\phi_{\mathrm{A}} \tag{10}$$

$$Z_2 = \eta_2 + \phi_{\mathrm{W}}/N_{\mathrm{A}} \tag{11}$$

$$Z_3 = 1 + \varepsilon^{-1} \tag{12}$$

and

$$\delta = (\alpha/\beta)^{0.5}$$

Taking the derivative of Π with respect to T_{WL} and setting it equal to zero $(\partial\Pi/\partial T_{\mathrm{WL}} = 0)$ gives

$$T_{\mathrm{WL,opt}}^n = \left[\delta T_{\mathrm{H}}^n/Z_3^{(n+1)/2} + T_{\mathrm{L}}^n\right]/\left[1 + \delta Z_3^{(n-1)/2}\right] \tag{13}$$

The corresponding warm refrigerant temperature is:

$$T_{\mathrm{WH,opt}}^n = \left[\delta T_{\mathrm{H}}^n Z_3^{(n-1)/2} + Z_3T_{\mathrm{L}}^n\right]/\left[1 + \delta Z_3^{(n-1)/2}\right] \tag{14}$$

Substituting equation (13) into equation (9) yields:

$$\Pi_{\mathrm{m}} = \alpha\phi_{\mathrm{A}}\left(T_{\mathrm{L}}^n - T_{\mathrm{H}}^n/Z_3^n\right)(Z_2 - Z_1Z_3)/\left(\delta + Z_3^{(1-n)/2}\right)^2 \tag{15}$$

Equation (15) is the main result of this paper. It determines the optimal profit for the given COP and the optimal COP for the given profit. It is called the finite-time exergoeconomic fundamental optimal relation or optimal profit versus COP characteristics.

Equation (15) indicates that profit is zero when $\varepsilon = \epsilon_c = (T_H/T_L - 1)^{-1}$ and $\varepsilon = (Z_2/Z_1 - 1)^{-1}$. Hence, there exists an extreme profit for the refrigerator. The maximum profit may be found by taking the derivative of Π_m with respect to ε and setting it equal to zero ($\partial\Pi_m/\partial\varepsilon = 0$). For maximum profit, the COP bound (ε_m) satisfies the equation

$$\begin{aligned}&\delta Z_1 T_L^n Z_4^{(3n+1)/2} + n Z_1 T_L^n Z_4^{n+1} - (n-1) Z_2 T_L^n Z_4^n \\ &\quad + (n-1)\delta Z_1 T_H^n Z_4^{(n+1)/2} - n\delta Z_2 T_H^n Z_4^{(n-1)/2} \\ &\quad - Z_2 T_H^n = 0 \end{aligned} \tag{16}$$

where

$$Z_4 = 1 + \varepsilon_m^{-1} \tag{17}$$

The COP (ϵ_m) is different from both the classical reversible COP bound (ε_C) and the finite-time thermodynamic COP bound (COP at the maximum cooling load, ϵ_R), and was termed as finite-time exergoeconomic COP bound. It is dependent on T_H, T_L, T_0, δ, n, and (ϕ_W/ϕ_A).

Note that for the process to be potentially profitable, the following relationship must exist: $0 < (\phi_W/\phi_A) < 1$, because one unit of work can give rise to at least one unit of exergy output. As the price of exergy becomes very large compared with the price of work, i.e., $\phi_W \ll \phi_A$, $(\phi_W/\phi_A) \Rightarrow 0$, and $T_0 = T_H$, equation (15) becomes:

$$\Pi_m = \phi_A \eta_2 R \tag{18}$$

where R is the optimal cooling load for the given COP [9, 13].

$$R = Q_2/\tau = \alpha\big(T_L^n - T_H^n/Z_3^n\big)/\big(\delta + Z_3^{(1-n)/2}\big)^2 \tag{19}$$

That is, the profit maximization approaches cooling load maximization.

On the other hand, as the price of exergy approaches the price of work, i.e., $(\phi_W/\phi_A) \Rightarrow 1$, equation (15) becomes:

$$\begin{aligned}\Pi_m = {}& -\phi_A \alpha T_0 \big(T_L^n - T_H^n/Z_3^n\big)(T_L Z_3 - T_H)/ \\ &\big[T_H T_L\big(\delta + Z_3^{(1-n)/2}\big)\big]^2\end{aligned} \tag{20}$$

The rate of entropy production of the refrigerator for the given COP is

$$\begin{aligned}\sigma &= \Delta S/\tau = R(Z_3/T_H - 1/T_L) \\ &= \alpha\big(T_L^n - T_H^n/Z_3^n\big)(T_L Z_3 - T_H) \\ &\quad /\big[T_H T_L\big(\delta + Z_3^{(1-n)/2}\big)\big]^2\end{aligned} \tag{21}$$

where ΔS is the change in entropy over the cycle.

Comparing equations (21) and (20) gives:

$$\Pi_m = -\phi_A T_0 \sigma = -\phi_A \Delta S/\tau \leqslant 0 \tag{22}$$

That is, the profit maximization approaches the rate of entropy production minimization,or in other words, the minimum waste of exergy ($T_0\Delta S$). Equation (22) indicates that the refrigerator is not profitable regardness of the COP at which the refrigerator is operating. Only if the refrigerator is operating reversibly ($\varepsilon = \varepsilon_C$) will the revenue equal the cost, and then the maximum profit will equal zero. (The corresponding rate of entropy production is also zero.)

Therefore, for any intermediate (ϕ_W/ϕ_A), the finite-time exergoeconomic performance bound (ε_m) lies between the finite-time thermodynamic performance bound and the reversible performance bound. ε_m is related to the latter two through the price ratio, and the associated COP bounds are the upper and lower limits of ε_m.

2.2. Optimal profit versus cop characteristics for three common heat transfer laws

The optimal profit versus COP characteristics is discussed in this section for three common heat transfer laws: Newton's Law ($n = 1$), the linear phenomenological law in irreversible thermodynamics ($n = -1$), and the radiative heat transfer law ($n = 4$).

Case $n = 1$

In this case, equation (15) becomes:

$$\Pi_m = \alpha\phi_A (T_L - T_H/Z_3)(Z_2 - Z_1 Z_3)/(1+\delta)^2 \tag{23}$$

The solution of equation (16) is:

$$\varepsilon_m = \big[\big(T_H Z_2/(T_L Z_1)\big)^{0.5} - 1\big]^{-1} \tag{24}$$

The maximum profit is:

$$\Pi_{max} = \alpha\phi_A\big[(T_H Z_2)^{0.5} - (T_L Z_1)^{0.5}\big]^2/(1+\delta)^2 \tag{25}$$

The corresponding cooling load is:

$$R_m = \alpha T_L\big[1 - \big(T_H Z_1/(T_L Z_2)\big)^{0.5}\big]/(1+\delta)^2 \tag{26}$$

As $T_0 = T_H$ and $(\phi_W/\phi_A) \Rightarrow 0$, then equation (23) becomes:

$$\Pi_m = \phi_A \eta_2 R \tag{27}$$

where

$$R = \alpha(T_L - T_H/Z_3)/(1+\delta)^2 \tag{28}$$

The cooling load (R) is a monotonic decreasing function of ε, and R approaches $R_{max} = \alpha T_L/(1+\delta)^2$ as ε approaches $\varepsilon_{min} = 0$ and R approaches $R_{min} = 0$ as ε approaches ε_c.

An interesting question is how to determine the profit for a given cooling load R or a given power input, $P_{in} = W/\tau$.

From equation (28),

$$\begin{aligned}\varepsilon = \big\{\big[(T_H - T_L + DP_{in})^2 + 4T_L DP_{in}\big]^{0.5} \\ - (T_H - T_L + DP_{in})\big\}/(2DP_{in}) = E/F\end{aligned} \tag{29}$$

and

$$\varepsilon = (T_L - DR)/(T_H - T_L + DR) \tag{30}$$

where $D = (1+\delta)^2/\alpha$.

Substituting equations (29) and (30) into equation (23) yields the optimal profit for the given power input (P_{in}) or for the given cooling load (R)

$$\begin{aligned}\Pi_m = \phi_A\big[T_L F - E(T_H - T_L)\big] \\ \times \big[(Z_2 - Z_1) - Z_1 F/E\big]/\big[D(E+F)\big]\end{aligned} \tag{31}$$

or

$$\Pi_m = \phi_A R\big[Z_2 - Z_1 T_H/(T_L - DR)\big] \tag{32}$$

Following the definition of two dimensionless parameters [6, 8]:

$$\begin{aligned}&(T_L/T_H)^\gamma = T_{WL}/T_{WH}, \\ &1 \leqslant \gamma \leqslant \infty\end{aligned} \tag{33}$$

$$\begin{aligned}&\theta = \big(T_{WL} - T_H^{1-\gamma}T_L^\gamma\big)/\big(T_L - T_H^{1-\gamma}T_L^\gamma\big), \\ &0 \leqslant \theta \leqslant 1\end{aligned} \tag{34}$$

Then, equations (9) and (15) become:

$$\begin{aligned}\Pi = \alpha\phi_A\big[Z_2 - Z_1(T_H/T_L)^\gamma\big] \\ /\big[(T_H/T_L)^\gamma\big[T_{WL}(T_H/T_L)^\gamma - T_H\big]^{-1} \\ - \delta^2(T_L - T_{WL})^{-1}\big]\end{aligned} \tag{35}$$

$$\begin{aligned}\Pi_m = \alpha\phi_A T_L\big[1 - (T_L/T_H)^{\gamma-1}\big] \\ \times \big[Z_2 - Z_1(T_H/T_L)^\gamma\big]/(1+\delta)^2\end{aligned} \tag{36}$$

Which obtains

$$\psi = \Pi/\Pi_m = (1+\delta)^2\theta(1-\theta)/\big[1 + (\delta^2 - 1)\theta\big] \tag{37}$$

Equation (37) indicates that $\psi = 1$ and $\Pi = \Pi_m$ when $\theta = \theta_A$

$$\theta_A = (1+\delta)^{-1} \tag{38}$$

and $\Pi_m = \Pi_{max}$ when $\gamma = \gamma_m$.

$$\gamma_m = \big[1 + \ln(Z_2/Z_1)/\ln(T_H/T_L)\big]/2 \tag{39}$$

Equations (35)–(39) are termed the profit and COP holographic spectra of Newton's Law for refrigeration systems. From the view of compromise optimization of profit and COP, we have

$$1 < \gamma \leqslant \gamma_m, \quad \text{and} \quad \theta = \theta_A \tag{40}$$

Equation (40) is termed as the finite-time exergoeconomic optimization criteria for Newton's Law refrigerators [10, 11].

As $(\phi_W/\phi_A) \Rightarrow 0$ and $T_0 = T_H$, equation (40) becomes:

$$1 < \gamma < \infty, \qquad Q = Q_A \tag{41}$$

Equation (41) is the main result of Sun [6] and Chen [8].

As $(\phi_W/\phi_A) \Rightarrow 1$, ε_m becomes ε_C, $\gamma_m \Rightarrow 1$, and the profit (Π_m) and the cooling load (R) approach zero. The optimal function of Π_m and R are schematically plotted in *figure 2* as functions of the COP (ε) and (ϕ_W/ϕ_A). For the case of $n = 4$, the optimal characteristics is similar to that of case of $n = 1$ as shown in *figure 2*.

Case $n = -1$

Note that in this case α and β are negative. The profit versus COP characteristics, the maximum profit, and the corresponding COP bounds are:

$$\begin{aligned}\Pi_m = -\alpha\phi_A\big[(Z_3 T_L - T_H)(Z_2 - Z_1 Z_3)\big] \\ /\big[T_H T_L(Z_3 + \delta)^2\big]\end{aligned} \tag{42}$$

$$\begin{aligned}\Pi_{max} = -\alpha\phi_A(Z_1 - Z_2 T_L/T_H)^2 \\ /\big[4T_L(\delta Z_1 + Z_2)(1 + \delta T_L/T_H)\big]\end{aligned} \tag{43}$$

and

$$\begin{aligned}\varepsilon_m = \big\{\big[\big[\delta\eta_1 + 2\eta_2 + (2+\delta)(\phi_W/\phi_A)\big]T_H + \delta Z_2 T_L\big] \\ /\big[Z_1 T_H + \big[2\delta\eta_1 + \eta_2 \\ + (2\delta+1)(\phi_W/\phi_A)\big]T_L\big] - 1\big\}^{-1}\end{aligned} \tag{44}$$

ε_m is dependent on δ. As $\delta = 1$, $\delta = 0$, and $\delta \Rightarrow \infty$, equation (44) becomes:

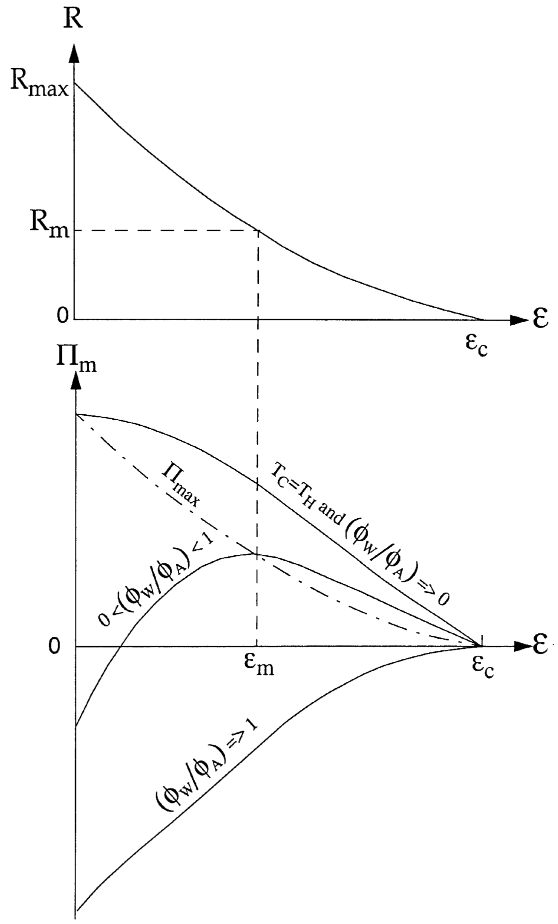

Figure 2. Optimal cooling load (R) and profit (Π_m) versus COP (ε) and (ϕ_W/ϕ_A) with $n = 1$.

$$\lim_{\delta=1} \varepsilon_m = \{[(\eta_1 + 2\eta_2 + 3(\phi_W/\phi_A))T_H + Z_2 T_L] / [Z_1 T_H + (2\eta_1 + \eta_2 + 3(\phi_W/\phi_A))T_L] - 1\}^{-1} \quad (45)$$

$$\lim_{\delta=0} \varepsilon_m = [2Z_2 T_H/(Z_1 T_H + Z_2 T_L) - 1]^{-1} \quad (46)$$

and

$$\lim_{\delta=\infty} \varepsilon_m = \{[(Z_1 T_H + Z_2 T_L)/(2Z_1 T_L)] - 1\}^{-1} \quad (47)$$

respectively.

The corresponding cooling load is:

$$R_m = -\alpha[(Z_5 + Z_6)Z_7]/[4T_H Z_8^2 (T_H + \delta T_L)] \quad (48)$$

$$Z_5 = [(1-\delta)\eta_1 - 2\eta_2 - (1+\delta)(\phi_W/\phi_A)]T_H \quad (49)$$

$$Z_6 = [2\delta\eta_1 + (1-\delta)\eta_2 + (1+\delta)(\phi_W/\phi_A)]T_L \quad (50)$$

$$Z_7 = [\delta\eta_1 + 2\eta_2 + (2+\delta)(\phi_W/\phi_A)]T_H + \delta Z_2 T_L \quad (51)$$

$$Z_8 = [\delta\eta_1 + \eta_2 + (1+\delta)(\phi_W/\phi_A)]T_H + \delta Z_2 T_L \quad (52)$$

As $(\phi_W/\phi_A) \Rightarrow 0$ and $T_0 = T_H$, equation (42) becomes:

$$\Pi_m = \phi_A \eta_2 R \quad (53)$$

where

$$R = \alpha(T_H - T_L Z_3)/[(T_H T_L)(Z_3 + \delta)^2] \quad (54)$$

The cooling load is a parabolic function of COP. The profit and cooling load approach their maximums when $\varepsilon = \varepsilon_R$.

$$\varepsilon_R = T_L/[2T_H - (1-\delta)T_L] \quad (55)$$

For $\delta = 1$, $\delta = 0$, and $\delta \Rightarrow \infty$, equation (55) becomes

$$\lim_{\delta=1} \varepsilon_R = T_L/(2T_H) \quad (56)$$

$$\lim_{\delta=0} \varepsilon_R = T_L/(2T_H - T_L) \quad (57)$$

and

$$\lim_{\delta=\infty} \varepsilon_R = 0 \quad (58)$$

respectively.

From equation (54),

$$\varepsilon = (Z_9 + Z_{10})/Z_{11} \quad (59)$$

$$Z_9 = \{[(2T_H T_L D P_{in})/(1+\delta) - (T_H - T_L)]^2 - 4DT_H T_L^2 P_{in}(T_H P_{in}/\alpha + 1)\}^{0.5} \quad (60)$$

$$Z_{10} = T_H - T_L - 2DP_{in} T_H T_L/(1+\delta) \quad (61)$$

$$Z_{11} = 2T_H T_L D P_{in} \quad (62)$$

and

$$\varepsilon = (Z_{12} - Z_{13})/Z_{14} \quad (63)$$

$$Z_{12} = [\alpha^2 T_L^2 + 4\alpha T_H T_L (T_H + \delta T_L)R]^{0.5} \quad (64)$$

$$Z_{13} = 2(1+\delta)T_H T_L + \alpha T_L \quad (65)$$

$$Z_{14} = 2[(1+\delta)^2 T_H T_L R - \alpha(T_H - T_L)] \quad (66)$$

Substituting equation (59) into equation (42) yields the optimal profit for the given power input (P_{in})

$$\Pi_m = \alpha\phi_A[(T_H - T_L)E_1 - T_L F_1][(\eta_2 - \eta_1)E_1 - Z_1 F_1] / [T_H T_L(\delta E_1 + F_1)^2] \quad (67)$$

where

$$E_1 = Z_9 + Z_{10} \quad (68)$$

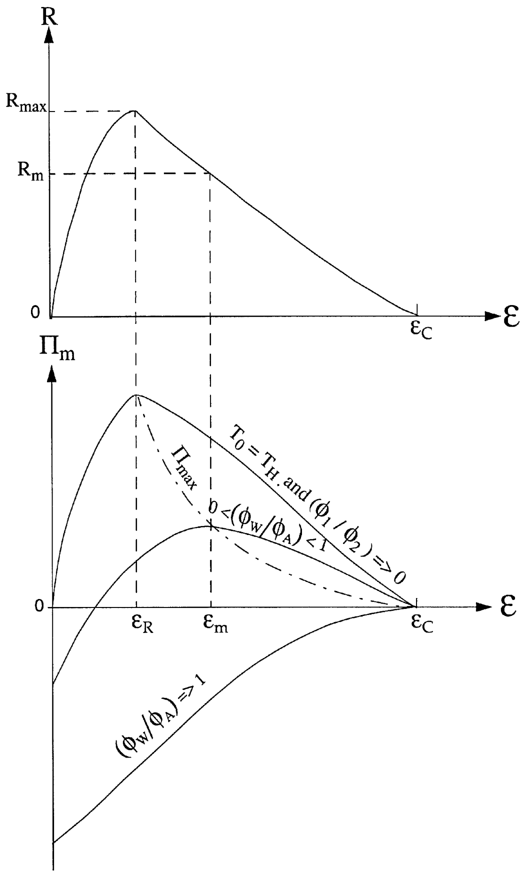

Figure 3. Opitmal cooling load (R) and profit (Π_{m}) versus COP (ε) and ($\phi_{\mathrm{W}}/\phi_{\mathrm{A}}$) with $n = -1$.

and

$$F_1 = 2DP_{\mathrm{in}}T_{\mathrm{H}}T_{\mathrm{L}} \tag{69}$$

Substituting equation (63) into equation (42) yields the optimal profit for the given cooling load (R)

$$\Pi_{\mathrm{m}} = \phi_{\mathrm{A}}R\left[Z_2 - Z_1(Z_{12} + Z_{14} - Z_{13})/(Z_{12} - Z_{13})\right] \tag{70}$$

The optimal function of Π_{m} and R are schematically plotted in *figure 3* as functions of COP ε.

Case $n = 4$

In this case, equation (15) becomes

$$\Pi_{\mathrm{m}} = \alpha\phi_{\mathrm{A}}\left(T_{\mathrm{L}}^4 - T_{\mathrm{H}}^4/Z_3^4\right)(Z_2 - Z_1Z_3)/\left(\delta + Z_3^{-3/2}\right)^2 \tag{71}$$

For maximum profit, the COP bound (ε_{m}) satisfies the following equation

$$\delta Z_1T_{\mathrm{L}}^4Z_4^{13/2} + 4Z_1T_{\mathrm{L}}^4Z_4^5 - 3Z_2T_{\mathrm{L}}^4Z_4^4 + 3\delta Z_1T_{\mathrm{H}}^4Z_4^{5/2} - 4\delta Z_2T_{\mathrm{H}}^4Z_4^{3/2} - Z_2T_{\mathrm{H}}^4 = 0 \tag{72}$$

where $Z_4 = 1 + \varepsilon_{\mathrm{m}}^{-1}$.

The maximum profit is

$$\Pi_{\mathrm{Max}} = -\phi_{\mathrm{A}}\alpha\left(T_{\mathrm{L}}^4 - T_{\mathrm{H}}^4/Z_4^4\right)(Z_2 - Z_21Z_4)/(\delta + Z_4^{-3/2})^2 \tag{73}$$

The corresponding cooling load is:

$$R_{\mathrm{m}} = \alpha\left(T_{\mathrm{L}}^4 - T_{\mathrm{H}}^4/Z_4^4\right)/\left(\delta + Z_4^{-3/2}\right)^2 \tag{74}$$

As $T_0 = T_{\mathrm{H}}$ and $(\phi_{\mathrm{W}}/\phi_{\mathrm{A}}) = 0$, equation (71) becomes:

$$\Pi_{\mathrm{m}} = \phi_{\mathrm{A}}\eta_2R \tag{75}$$

where

$$R = \alpha\left(T_{\mathrm{L}}^4 - T_{\mathrm{H}}^4/Z_3^4\right)/\left(\delta + Z_3^{-3/2}\right)^2 \tag{76}$$

The cooling load (R) is a monotonic decreasing function of ε, and R approaches $R_{\max} = \alpha T_{\mathrm{L}}^4/\delta$ as ε approaches $\varepsilon_{\min} = 0$ and R approaches $R_{\min} = 0$ as ε approaches ε_{c}.

The optimal function of Π_{m} and R are schematically plotted in *figure 2* as functions of the cop (ε), which are analogy to those in the case of $n = 1$.

3. CONCLUSION

This paper derives the optimal profit and efficiency characteristics and the maximum profit and its corresponding COP bound of an endoreversible refrigerator based on a relatively general heat transfer law. We seek the economic optimization objective function instead of pure thermodynamic parameters by viewing the refrigerator as a production process. It is shown that the economic and thermodynamic optimization converged in the limits $(\phi_{\mathrm{W}}/\phi_{\mathrm{A}}) \Rightarrow 0, 1$. When the profit margin for exergy conversion is small, the maximum profit operation is near the minimum loss of exergy operation, while when the work is very cheap compared to the price of energy, the maximum profit operation is near the maximum cooling load operation. For intermediate $(\phi_{\mathrm{W}}/\phi_{\mathrm{A}})$, any publically desirable COP can be made profit-optimal by regulating the ratio $(\phi_{\mathrm{W}}/\phi_{\mathrm{A}})$ of input and output prices. The study of this paper is the combination of endoreversible cycle and economic optimization. We call it as finite-time exergoeconomic analysis. It is worthwhile to note that the finite-time exergoeconomic COP bound always exists for any heat transfer law while the finite-time thermodynamic COP bound only exists for $n < 1$.

It is important to investigate the effect of heat transfer law on the finite time exergoeconomic performance of a Carnot refrigerator. This paper provides a generalized heat transfer law $q \propto \Delta(T^n)$. When $n = -1$, the heat transfer law is the linear phenomenological law in irreversible thermodynamics. The coefficients α and β are the so-called kinetic coefficients. When $n = 1$, the heat transfer law is Newton's heat transfer law. When $n = 2$, the heat transfer law is applicable to radiation propagated along a one-dimensional transmission line. The coefficients α is equal to $[\pi^2 k^2/(6h)]$, where h is the Planck's constant and k is the Stefan–Boltzmann constant. When $n = 3$, the heat transfer law is applicable to radiation propagated along a two-dimensional surface. When $n = 4$, the heat transfer law is a model for a radiant refrigerator with radiative heat transfer. Heat is removed from the warm working fluid to the heat sink by means of radiation. Heat is also radiated back from the heat sink to the warm working fluid by Kirchhoff's law. Assuming black bodies in the radiative heat transfer, the Stefan–Boltzmann law requires that the radiative heat transfer is proportional to the fourth power of the body temperatures. The coefficients ∀ and ∃ are related to the Stefan-Boltzmann constant. Therefore, the results of this paper are universal for the performance analysis and optimization of Carnot refrigerators.

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