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# Ejercicios de Transformadas

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## Ejercicios de Transformadas.

### Fracciones Parciales:

$$\checkmark \quad 1 / s (s^2 + 2) = A/s + Bs + C / s^2 + 2$$

$$1 = A (s^2 + 2) + s (Bs + c)$$

$$1 = As^2 + 2A + Bs^2 + Cs$$

$$1 = (A+B) s^2 + Cs + 2A$$

$$A+B = 0$$

$$C=0$$

$$2A=1$$

$$\frac{1}{2} + B = 0$$

$A = 1/2$
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$B = -1/2$
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$C = 0$
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$$1 / s (s^2 + 2) = \underline{1/2 / s - 1/2 s + 0 / s^2 + 2}$$

$$\checkmark \quad 3 - s / s (s^2 + 1) = A/s + Bs + C / s^2 + 1$$

$$3 - s = A (s^2 + 1) + s (Bs + C) / s^2 + 1$$

$$3 - s = As^2 + A + Bs^2 + Cs$$

$$3 - s = (A+B) s^2 + Cs + A$$

$$A+B = 0$$

$$C = -1$$

$$A=3$$

$$3 + B = 0$$

$A = 3$
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$B = -3$
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$C = -1$
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$$3 - s / s (s^2 + 1) = \underline{3/s - 3s - 1 / s^2 + 1}$$

$$\checkmark \quad 3s + 1 / s^2 (s^2 + 25) = As + B / s^2 + Cs + D / s^2 + 25$$

$$3s + 1 = (s^2 + 25)(As + B) + s^2 (Cs + D)$$

$$3s + 1 = s^2A + Bs^2 + 25As + 25B + s^2C + Ds^2$$

$$3s + 1 = (A+C)s^3 + (B+D)s^2 + 25As + 25B$$

$$A+C = 0$$

$$B+D = 0$$

$$25A = 3$$

$$25B = 1$$

$$A = 3 / 25$$

$$B = 1 / 25$$

A+C=0 sustituyendo el valor de A queda:  
 $3/25 + C = 0$

$$C = -3 / 25$$

B + D = 0 sustituyendo el valor de B queda  
 $1/25 + D = 0$

$$D = -1 / 25$$

$$3s + 1 / s^2 (s^2 + 25) = \underline{\underline{3/25 s + 1/25 / s^2 - 3/25s - 1/25 / s^2 + 25}}$$

$$\checkmark \quad 1 / s^4 - 16 = A / (s - 2) + B / (s+2) + Cs + D / s^4 - 16$$

$$1 = A (s+2)(s^4 - 4) + B (s-2)(s^2 - 4) + (Cs + D)(s - 2)(s + 2)$$

Cuando s=2

$$1 = A(4)(8) + 0 + 0$$

$$1 = 32A$$

$$A = 1/32$$

Cuando s = -2

$$1 = 0 + B(-4)(8) + 0$$

$$1 = -32A$$

$$B = -1/32$$

Desarrollando:

$$1 = (As + 2A)(s^4 + 4) + (Bs - 2B)(s^4 + 4) + (Cs + D)(s^2 - 4)$$

$$1 = As^3 + 4As + 2As^2 + 8A + Bs^3 + 4Bs - 2Bs^2 - 8B + Cs^3 - 4Cs + Ds^2 - 4D$$

$$1 = (A+B+C)s^3 + (2A - 2B + D)s^2 + (4A + 4B - 4C)s + (8A - 8B - 4D)$$

1.  $A+B+C=0$
2.  $2A - 2B + D=0$
3.  $4A + 4B - 4C = 0$
4.  $8A - 8B - 4D=1$

De la 1 sustituimos:

$$1/32 - 1/32 + C = 0$$

$C = 0$
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$$2 * 1/32 - 2(-1/32) + D = 0$$

$$2/32 + 2/32 + D = 0$$

$$4/32 + D = 0$$

$D = -4/32$
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$$1/s^4 - 16 = \underline{1/32 / (s - 2) - 1/32 / (s+2) - 4/32 / s^4 - 16}$$

## Números Complejos:

### Parte A

$$\checkmark \quad 5i/2 + i = 1 + 2i$$

$$5i/2 + i * 2 - i/2 - i = (10i + 5/4 + 1) = (10i + 5/5) = 10i/5 + 5/5 = 2i + 1 = \underline{1+2i}$$

$$\checkmark \quad (-1+i)^7 = -8(1+i)$$

$$[-1+i] = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \arctg^{-1} 1 = \text{phi} / 4$$

$$\theta = \text{phi} - \text{phi} / 4 = 3\text{phi} / 4$$

$$(-1+i)^7 = \sqrt{2} (\cos(3\text{phi}/4) + i \text{sen}(3\text{phi}/4))^7$$

$$(-1+i)^7 = \sqrt{(2)^7} (\cos(3\text{phi}/4)^7 + i \text{sen}(3\text{phi}/4)^7)$$

$$(-1+i)^7 = 8\sqrt{2} (\cos(21\text{phi}/4) + i \text{sen}(21\text{phi}/4)^7)$$

$$(-1+i)^7 = 8\sqrt{2} (-\cos(21\text{phi}/4) - i \text{sen}(21\text{phi}/4))$$

$$(-1+i)^7 = 8\sqrt{2} (-\sqrt{2}/2 - i\sqrt{2}/2)$$

$$(-1+i)^7 = 8(\sqrt{4}/2 + i\sqrt{4}/2)$$

$$(-1+i)^7 = 8(2/2 + i 2/2)$$

$$(-1+i)^7 = \mathbf{8(1+i)}$$

$$\begin{aligned}
 \checkmark \quad & i(1 - i\sqrt{3})(\sqrt{3} + 1) = 2 + 2i\sqrt{3} \\
 & i(1 + i\sqrt{3})(\sqrt{3} + 1) \\
 & i\sqrt{3} + i^2 + \sqrt{9} + i\sqrt{3} \\
 & i\sqrt{3} - 1 + 3 + i\sqrt{3} \\
 & \underline{\underline{2 + 2i\sqrt{3}}}
 \end{aligned}$$

$$\checkmark \quad (1 + i\sqrt{3})^{-10} - 2^{-11} = (-1 + i\sqrt{3})$$

$$\begin{aligned}
 (1 + i\sqrt{3})^{-10} &= 1 / (1 + i\sqrt{3})^{10} \\
 [(1 + i\sqrt{3})] &= \sqrt{1 + 3} = \sqrt{4} = 2 \\
 \theta &= \arctg^{-1} \sqrt{3} = \text{phi} / 3 \\
 (1 + i\sqrt{3})^{-10} &= 1 / (2 \text{ cis } \text{phi} / 3)^{10} = 1 / (2^{10} \text{ cis } 10\text{phi} / 3) \\
 &= 1 / 2^{10} (\cos \text{phi} / 3 + i \text{sen } \text{phi} / 3) \\
 &= 1 / (2^{10} (-1/2 + i\sqrt{3} / 2)) \\
 &= 1 / (2^{10} (-1/2 + i\sqrt{3} / 2) * (-1/2 + i\sqrt{3} / 2) / (-1/2 + i\sqrt{3} / 2)) \\
 &= 1 / (2^{10} (1/4 + 3/4) * (-1/2 + i\sqrt{3} / 2) / 2^{10} * 1) \\
 (1 / 2) * 2^{10} (-1/2 + i\sqrt{3} / 2) / 1 &= (2^{-10} / 2) * (-1/2 + i\sqrt{3} / 2) = \mathbf{2^{-11} (-1 + i\sqrt{3})}
 \end{aligned}$$

## Parte B

$$\checkmark [z-1-i] = 1$$

$$[(x+iy) - 1 - i] = 1$$

$$[x-1 + i(y-1)] = 1$$

$$\sqrt{(x-1)^2 + (y-1)^2} = 1$$

$$(x-1)^2 + (y-1)^2 = \underline{1}$$

$$\checkmark [z-1] = [z+i]$$

$$[(x+iy) - 1] = [(x+iy) + i]$$

$$[(x-1) + iy] = [(x+i)(y+1)]$$

$$\sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y+1)^2}$$

$$x^2 - 2x + 1 + y^2 = x^2 + y^2 + 2y + 1$$

$$-2x + 1 = 2y + 1$$

$$-2x - 2y + 1 - 1 = 0$$

$$-2x - 2y = 0$$

$$\underline{\mathbf{x=y}}$$

$$\checkmark [z-1+i] \neq [z-1-i]$$

$$[x+iy-1+i] \neq [x+iy-1-i]$$

$$[(x-1) + i(y+1)] = [(x-1) + i(y-1)]$$

$$\sqrt{(x-1)^2 + (y+1)^2} \neq \sqrt{(x-1)^2 + (y-1)^2}$$

$$(y+1)^2 \neq (y-1)^2$$

$$y^2 + 2y + 1 \neq y^2 - 2y + 1$$

$$2y \neq -2y$$

$$2y + 2y \neq 0$$

$$4y \neq 0$$

$$y \neq 0$$

$$\checkmark \cos(z-i) = 2$$

$$\cos^{-1}(2) = z-1$$

$$z = \cos^{-1}(2) + i$$

Por formula:

$$\cos^{-1}(z) = [-i \log(z + i(1-z^2)^{1/2})]$$

$$z = [-i \log(2 + i(1-z^2)^{1/2})] + i + 2\pi n$$

$$z = [i - i \log(2 + i(-3^2)^{1/2})] + i + 2\pi n$$

$$z = [i - i \log(2 + i(\sqrt{3}i)^{1/2})] + 2\pi n$$

$$z = [i - i \log(2 +/\!-\!(\sqrt{3})^{1/2})] + 2\pi n$$

$$z = \underline{\mathbf{2\pi n + i [1 - \log(2 +/\!-\!(\sqrt{3})^{1/2})]}}$$

✓ Encontrar la ecuación de la circunferencia que pasa por los puntos  $z_1=1-i$ ,  $z_2=2i$  y  $z_3=1+i$

La ecuación de la circunferencia es la siguiente:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Los números complejos en su forma par:

$$Z_1 = (1, -1) = 1 + 1 + D + E + F = 0$$

$$Z_2 = (0, 2) = 0 + 4 + 0 + 2E + F = 0$$

$$Z_3 = (1, 1) = 1 + 1 + D + E + F = 0$$

$$D + E + F = -2$$

$$2E + F = -4$$

$$D + E + F = -2$$

Resolver por sistema de Gauss, el resultado es el siguiente:

$$X^2 + y^2 + 2x - 4 = 0$$

$$(X^2 + 2x) + y^2 = 4$$

$$(X^2 + 2x + 1) + y^2 = 4 + 1$$

$$\underline{(X^2 + 1) y^2 = 5}$$