1)

$$\int \frac{y^4 + 2y^2 - 1}{\sqrt{y}}\,dy = \int \frac{y^4 + 2y^2 - 1}{y^{1/2}}\,dy = \int y^{7/2} + 2y^{3/2} - y^{-\frac{1}{2}}\,dy$$

$$= \frac{2}{9}y^{\frac{9}{2}} + \frac{4}{5}y^{\frac{5}{2}} - 2y^{\frac{1}{2}} + C$$

$$\int \frac{y^4 + 2y^2 - 1}{\sqrt{y}}\,dy = \frac{2}{9}y^{\frac{9}{2}} + \frac{4}{5}y^{\frac{5}{2}} - 2y^{\frac{1}{2}} + C$$

2)

$$\int \frac{\sin x}{\cos^2 x}\,dx = \qquad z = \cos x, \qquad\qquad dz = -\sin x\ dx$$

$$\int \frac{\sin x}{\cos^2 x}\,dx = -\int \frac{dz}{z^2} = \frac{1}{z} + C = \frac{1}{\cos x} + C = \sec x + C$$

$$\int 4\cosec x\ \cotan x + 2\sec^2 x\ dx = \int \left(\frac{4}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) + 2\sec^2 x\ dx$$

$$= 4\int \frac{\cos x}{\sin^2 x}\,dx + 2\int \sec^2 x\ dx$$

$$4\int \frac{\cos x}{\sin^2 x}\,dx = \qquad z = \sin x \qquad\qquad dz = \cos x\ dx$$

$$4\int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{4}{z^2}\,dz = \frac{-4}{z} + C = -4\frac{1}{\sin x} + C = -4\cosec x + C$$

$$\int 4\cosec x\ \cotan x + 2\sec^2 x\ dx = -4\,\cosec x + 2\tan x + C$$

3)

$$\int 3\,cosec^2x - 5\sec x \tan x\,dx = 3\int cosec^2x\,dx - 5\int \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)dx =$$

$$3\int cosec^2x\,dx - 5\int \left(\frac{\sin x}{\cos^2 x}\right)dx = -3\,cotan\,x - 5\int \left(\frac{\sin x}{\cos^2 x}\right)dx =$$

$$5\int \left(\frac{\sin x}{\cos^2 x}\right)dx = z = \cos x \qquad\qquad dz = -\sin x\,dx$$

$$-5\int \frac{dz}{z^3} = \frac{5}{2}\left(\frac{1}{z^2}\right) + C = \frac{5}{2}\frac{1}{\cos^2 x} + C$$

$$3\int cosec^2x\,dx - 5\int \left(\frac{\sin x}{\cos^2 x}\right)dx = -3\,cotan\,x - \frac{5}{2}\frac{1}{\cos^2 x} + C$$

$$\int 3\,cosec^2x - 5\sec x \tan x\,dx = -3\,cotan\,x - \frac{5}{2}\frac{1}{\cos^2 x} + C$$

4)

$$\int x^2(x^3 - 1)^{10}dx = \qquad z = x^3 - 1, \qquad\qquad dz = 3x^2dx$$

$$\int x^2(x^3 - 1)^{10}dx = \frac{1}{3}\int z^{10}dz = \frac{1}{3}\left(\frac{1}{11}\right)z^{11} + C = \frac{1}{33}(x^3 - 1)^{11} + C$$

$$\int x^2(x^3 - 1)^{10}dx = \frac{1}{33}(x^3 - 1)^{11} + C$$

5)

$$\int \frac{2r}{(1-r)^7} \, dr = \quad z = 1-r \qquad r = 1-z$$

$$dz = -dr$$

$$\int \frac{2r}{(1-r)^7} \, dr = -2\int \frac{1-z}{z^7} \, dz = -2\int \frac{1}{z^7} - \frac{1}{z^6} \, dz = -\frac{1}{3}\left(\frac{1}{z^6}\right) + \frac{2}{5}\left(\frac{1}{z^5}\right) + C$$

$$\int \frac{2r}{(1-r)^7} \, dr = -\frac{1}{3}\left(\frac{1}{(1-r)^6}\right) + \frac{2}{5}\left(\frac{1}{(1-r)^5}\right) + C$$

$$\boxed{\int \frac{2r}{(1-r)^7} \, dr = -\frac{1}{3}\left(\frac{1}{(1-r)^6}\right) + \frac{2}{5}\left(\frac{1}{(1-r)^5}\right) + C}$$

6)

$$\int \sin 2x \sqrt{2 - \cos 2x} \, dx = \quad z = 2 - \cos 2x, \qquad dz = 2\sin 2x \, dx$$

$$\int \sin 2x \sqrt{2 - \cos 2x} \, dx = \frac{1}{2}\int \sqrt{z} \, dz = \frac{1}{2}\frac{2}{3}z^{3/2} + C = \frac{1}{3}(2 - \cos 2x)^{3/2} + C$$

$$\boxed{\int \sin 2x \sqrt{2 - \cos 2x} \, dx = \frac{1}{3}(2 - \cos 2x)^{3/2} + C}$$

7)

$$\int \frac{dx}{2 + e^x} = \int \frac{e^x dx}{e^x(2 + e^x)} = \int \frac{e^x dx}{2e^x + e^{2x}} = \quad z = e^x \qquad dx = e^x dx$$

$$\int \frac{e^x dx}{2e^x + e^{2x}} = \int \frac{dz}{z^2 + 2z} = \int \frac{dz}{z^2 + 2z + 1 - 1} = \int \frac{dz}{(z+1)^2 - 1}$$

$$\sin \delta = z + 1 \Rightarrow z = \sin \delta - 1$$

$$dz = \cos \delta \, d\delta \qquad \qquad \sqrt{(z+1)^2 - 1} = \cos \delta$$

$$(z+1)^2 - 1 = \cos^2 \delta$$

$$\int \frac{dz}{(z+1)^2 - 1} = \int \left(\frac{1}{\cos^2 \delta}\right) \cos \delta \ d\delta = \int \frac{d\delta}{\cos \delta} = \ln|\sec \delta + \tan \delta| + C$$

$$\ln \left|\frac{1}{\sqrt{(z+1)^2 - 1}} + \frac{z+1}{\sqrt{(z+1)^2 - 1}}\right| + C = \ln \left|\frac{z+2}{\sqrt{(z+1)^2 - 1}}\right| + C = \ln \left|\frac{e^x + 2}{\sqrt{(e^x + 1)^2 - 1}}\right| + C$$

$$\int \frac{dx}{2 + e^x} = \ln \left|\frac{e^x + 2}{\sqrt{(e^x + 1)^2 - 1}}\right| + C$$

8)

$$\int x^2 e^{2x} dx = \qquad z = x^2 \qquad\qquad dv = e^{2x} dx$$

$$dz = 2x dx \qquad\qquad v = \frac{e^{2x}}{2}$$

$$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \left(\int x e^{2x} dx\right) =$$

$$z = x \qquad\qquad dv = e^{2x} dx$$

$$dz = dx \qquad\qquad v = \frac{e^{2x}}{2}$$

$$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \left(\frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx\right) = \frac{1}{2} x^2 e^{2x} - \left(\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}\right) + C$$

$$\int x^2 e^{2x} dx = \frac{e^{2x}}{2} \left(x^2 - x + \frac{1}{2}\right) + C$$

9)

$$\int \frac{dx}{\sqrt{1 - 16x^2}} = \qquad\qquad \sin \delta = 4x \qquad\qquad \Rightarrow \delta = \arcsin 4x \qquad\qquad dx = \frac{1}{4} \cos \delta \ d\delta$$

$$\sqrt{1 - 16x^2} = \cos \delta$$

$$\int \frac{dx}{\sqrt{1-16x^2}} = \frac{1}{4}\int \left(\frac{1}{\cos\delta}\right)(\cos\delta)d\delta = \frac{1}{4}\int d\delta = \frac{\delta}{4} + C = \frac{1}{4}\arcsin 4x + C$$

$$\boxed{\int \frac{dx}{\sqrt{1-16x^2}} = \frac{1}{4}\arcsin 4x + C}$$

10)

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+1-1}} = \int \frac{dx}{\sqrt{1-(1-x)^2}}$$

$$\cos\phi = 1-x \qquad\qquad \Rightarrow \phi = \arccos(1-x) \qquad\qquad \Rightarrow x = 1-\cos\phi$$

$$dx = \sin\phi\, d\phi \qquad\qquad \sqrt{1-(1-x)^2} = \sin\phi$$

$$\int \frac{dx}{\sqrt{1-(1-x)^2}} = \int \left(\frac{1}{\sin\phi}\right)\sin\phi\, d\phi = \int d\phi = \phi + C = arccos(1-x) + C$$

$$\boxed{\int \frac{dx}{\sqrt{2x-x^2}} = arccos(1-x) + C}$$

11)

$$\int x3^x\, dx = \qquad\qquad z = x \qquad\qquad dv = 3^x dx$$

$$dz = dx \qquad\qquad v = \frac{3^x}{\ln 3}$$

$$\int x3^x\, dx = \frac{x3^x}{\ln 3} - \left(\int \frac{3^x}{\ln 3}dx\right) = \frac{x3^x}{\ln 3} - \frac{3^x}{\ln^2 3} + C$$

$$\boxed{\int x3^x\, dx = \frac{3^x}{\ln 3}\left(x - \frac{1}{\ln 3}\right) + C}$$

12)

$$\int x \sec^2 x \, dx = \qquad z = x \qquad dv = \sec^2 x dx$$

$$dz = dx \qquad v = \tan x$$

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx = x \tan x + \ln|\cos x| + C$$

$$\int x \sec^2 x \, dx = x \tan x + \ln|\cos x| + C$$

13)

$$\int \ln(x^2 + 1) dx = \qquad z = \ln(x^2 + 1) \qquad dv = dx$$

$$dz = \frac{2x}{x^2 + 1} dx \qquad v = x$$

$$\int \ln(x^2 + 1) dx = x \ln(x^2 + 1) - \left(\int \frac{2x^2}{x^2 + 1} dx \right)$$

$$\int \frac{2x^2}{x^2 + 1} dx = \qquad x = \tan \theta \qquad \Rightarrow \quad \theta = \arctan x$$

$$dx = \sec^2 \theta \, d\theta \qquad \sqrt{x^2 + 1} = \sec \theta \qquad \Rightarrow x^2 + 1 = \sec^2 \theta$$

$$\int \frac{2x^2}{x^2 + 1} dx = 2 \int \frac{\tan^2 \theta}{\sec^2 \theta} \sec^2 \theta \, d\theta = 2 \int \tan^2 \theta \, d\theta = 2(\tan \theta - \theta) + C$$

$$2(x - \arctan x) + C$$

$$\int \ln(x^2 + 1) dx = x \ln(x^2 + 1) - 2(x - \arctan x) + C$$

14)

$$\int x^2 \sin 3x = \qquad z = x^2 \qquad dv = \sin 3x \, dx$$

$$dz = 2x dx \qquad v = -\frac{1}{3}\cos 3x$$

$$\int x^2 \sin 3x = -\frac{1}{3}x^2 \cos 3x + \frac{2}{3}\left(\int x \, \cos 3x\right) dx =$$

$$\int x \, \cos 3x = \qquad z = x \qquad dv = \cos 3x \, dx$$

$$dx = dx \qquad v = \frac{1}{3}\sin 3x$$

$$\int x \, \cos 3x = \frac{1}{3}x \sin 3x - \frac{1}{3}\int \sin 3x \, dx = \frac{1}{3}x \sin 3x - \frac{1}{9}\cos 3x + C$$

$$\boxed{\int x^2 \sin 3x = -\frac{1}{3}x^2 \cos 3x + \frac{2}{3}\left(\frac{1}{3}x \sin 3x - \frac{1}{9}\cos 3x\right) + C}$$

15)

$$\int \sin x \ln|\cos x| \, dx = \qquad z = \cos x \qquad dz = -\sin x \, dx$$

$$-\int \ln z \, dz = \qquad u = \ln z \qquad dv = dx$$

$$du = \frac{dz}{z} \qquad v = z$$

$$-\int \ln z \, dz = -z \ln z + \int z \frac{dz}{z} = -z \ln z + z + C = -\cos x \left(\ln(\cos x) - 1\right) + C$$

$$\boxed{\int \sin x \ln|\cos x| \, dx = -\cos x \left(\ln(\cos x) - 1\right) + C}$$

16)

$$\int \frac{\sin 2x}{e^x}\,dx = \int e^{-x}\sin 2x\,dx = \qquad\qquad z = \sin 2x \qquad\qquad dv = e^{-x}dx$$

$$dz = 2\cos 2x\,dx \qquad\qquad v = -e^{-x}$$

$$\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x + 2\left(\int e^{-x}\cos 2x\,dx\right)$$

$$z = \cos 2x \qquad\qquad dv = e^{-x}\,dx$$

$$dz = 2\sin 2x\,dx \qquad\qquad v = -e^{-x}$$

$$\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x + 2\left(-e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\,dx\right)$$

$$\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x - 2e^{-x}\cos 2x - 4\int e^{-x}\sin 2x\,dx$$

$$(4+1)\int e^{-x}\sin 2x\,dx = -e^{-x}(\sin 2x + 2\cos 2x)$$

$$\int e^{-x}\sin 2x\,dx = \frac{-e^{-x}}{5}(\sin 2x + 2\cos 2x) + C$$

$$\boxed{\int \frac{\sin 2x}{e^x}\,dx = \frac{-e^{-x}}{5}(\sin 2x + 2\cos 2x) + C}$$

17)

$$\int \frac{e^{2x}}{\sqrt{1-e^x}}\,dx = \int \frac{e^x\, e^x}{\sqrt{1-e^x}}\,dx = \qquad z = 1 - e^x \qquad \Rightarrow e^x = 1 - z$$

$$dz = -e^x\,dx$$

$$-\int \frac{1-z}{\sqrt{z}}\,dz = \int \frac{z-1}{\sqrt{z}}\,dz = \int z^{\frac{1}{2}} - z^{-\frac{1}{2}}dz = \frac{2}{3}z^{\frac{3}{2}} - 2\,z^{\frac{1}{2}} + C$$

$$= \frac{2}{3}(1-e^x)^{\frac{3}{2}} - 2\,(1-e^x)^{\frac{1}{2}} + C$$

$$\boxed{\int \frac{e^{2x}}{\sqrt{1-e^x}}\,dx = \frac{2}{3}\sqrt{(1-e^x)^3} - 2\sqrt{1-e^x} + C}$$

18)

$$\int \cos^2\left(\frac{x}{2}\right) dx = \frac{1}{2}\int 1 + \cos x \; dx = \frac{1}{2}(x + \sin x) + C$$

$$\int \cos^2\left(\frac{x}{2}\right) dx = \frac{1}{2}(x + \sin x) + C$$

19)

$$\int \sqrt{\cos x}\, \sin^3 x \; dx = \int \sqrt{\cos x}\, \sin^2 x \sin x \; dx = \int \sqrt{\cos x}\,(1 - \cos^2 x) \sin x \; dx$$

$$\int (\sqrt{\cos x} - \cos x) \sin x \; dx = \qquad z = \cos x \qquad\qquad dz = -\sin x \; dx$$

$$-\int \sqrt{z} - z \; dz = \int z - \sqrt{z} \; dz = \frac{z^2}{2} - \frac{2}{3}\sqrt{z^3} + C = \frac{\cos^2 x}{2} - \frac{2}{3}\sqrt{\cos^3 x} + C$$

$$\int \sqrt{\cos x}\, \sin^3 x \; dx = \frac{\cos^2 x}{2} - \frac{2}{3}\sqrt{\cos^3 x} + C$$

20)

$$\int \sin(\ln(x)) \; dx = \int \sin(\ln(x))\frac{x}{x} dx = \qquad\qquad z = \ln x \qquad \Rightarrow x = e^z$$

$$dz = \frac{dx}{x}$$

$$\int \sin(\ln(x)) \; dx = \int e^z \sin z \; dz$$

$$\int e^z \sin z \; dz = \qquad u = \sin z \qquad\qquad dv = e^z dz$$

$$du = \cos x \; dx \qquad\qquad\qquad v = e^z$$

$$\int e^z \sin z \, dz = e^z \sin z - \left(\int e^z \cos z \, dz \right) =$$

$$\int e^z \cos z \, dz = \qquad u = \cos z \qquad\qquad dv = e^z \, dz$$

$$du = -\sin z \, dz \qquad\qquad v = e^z$$

$$\int e^z \sin z \, dz = e^z \sin z - \left(e^z \cos z + \int e^z \sin z \, dz \right) =$$

$$\int e^z \sin z \, dz = e^z \sin z - e^z \cos z - \int e^z \sin z \, dz =$$

$$(1+1) \int e^z \sin z \, dz = e^z \sin z - e^z \cos z =$$

$$\int e^z \sin z \, dz = \frac{e^z}{2} (\sin z - \cos z) + C = \frac{x}{2} (sin(\ln x) - cos(\ln x)) + C$$

$$\boxed{\int \sin(\ln(x)) \, dx = \frac{x}{2} (sin(\ln x) - cos(\ln x)) + C}$$

21)

$$\int e^x \tan^2(e^x) \, dx = \qquad z = e^x \qquad\qquad dz = e^x \, dz$$

$$\int \tan^2 z \, dz = \int \sec^2 z - 1 \, dz = \tan z - z + C = \tan e^x - e^x + C$$

$$\boxed{\int e^x \tan^2(e^x) \, dx = tan\, e^x - e^x + C}$$

22)

$$\int \frac{\sec^4(\ln(x))}{x} \, dx = \qquad\qquad z = \ln x \qquad\qquad dz = \frac{dx}{x}$$

$$\int \sec^4 z \, dz = \int \sec^2 z \sec^2 z \, dz = \int \sec^2 z \, (\tan^2 z + 1) dz =$$

$$\int \sec^2 z \tan^2 z \, dz + \int \sec^2 z \, dz = \left(\int \sec^2 z \, \tan^2 z \, dz \right) + \tan z$$

$$\int \sec^2 z \, \tan^2 z \, dz = \qquad u = \tan z \qquad\qquad du = \sec^2 z \, dz$$

$$\int \sec^2 z \, \tan^2 z \, dz = \int u^2 \, du = \frac{1}{3} u^3 + C = \frac{1}{3} \tan^3 z + C$$

$$\int \frac{\sec^4 (\ln(x))}{x} \, dx = \int \sec^4 z \, dz = \frac{1}{3} \tan^3 z + \tan z + C = \frac{1}{3} \tan^3 (\ln x) + \tan(\ln x) + C$$

$$\boxed{\int \frac{\sec^4 (\ln(x))}{x} \, dx = \frac{1}{3} \tan^3 (\ln x) + \tan(\ln x) + C}$$

23)

$$\int \frac{dx}{1 + \cos x} = \int \frac{1}{1 + \cos x} \frac{1 - \cos x}{1 - \cos x} \, dx = \int \frac{1 - \cos x}{1 - \cos^2 x} \, dx = \int \frac{1 - \cos x}{\sin^2 x} \, dx$$

$$\int \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} \, dx = \int \cosec^2 x \, dx - \int \frac{\cos x}{\sin^2 x} \, dx = -\cotan x - \left(\int \frac{\cos x}{\sin^2 x} \, dx \right)$$

$$\int \frac{\cos x}{\sin^2 x} \, dx = \qquad z = \sin x \qquad\qquad dz = \cos x \, dx$$

$$\int \frac{\cos x}{\sin^2 x} \, dx = \int \frac{dz}{z^2} = -\frac{1}{z} + C = -\frac{1}{\sin x} + C$$

$$\int \cosec^2 x \, dx - \int \frac{\cos x}{\sin^2 x} \, dx = -\cotan x + \frac{1}{\sin x} + C = -\cotan x + \cosec x + C$$

$$\boxed{\int \frac{dx}{1 + \cos x} = -\cotan x + \cosec x + C}$$

24)

$$\int \frac{\sec^2 x}{(4 - \tan^2 x)^{3/2}} \, dx = \int \frac{\sec^2 x}{\sqrt{(4 - \tan^2 x)^3}} \, dx = \qquad z = \tan x \qquad dz = \sec^2 x \, dx$$

$$\int \frac{\sec^2 x}{\sqrt{(4 - \tan^2 x)^3}} \, dx = \int \frac{dz}{\sqrt{(4 - z^2)^3}} = \qquad \cos \lambda = \frac{z}{2} \qquad \Rightarrow \lambda = \arccos\left(\frac{z}{2}\right)$$

$$z = 2\cos \lambda \qquad\qquad dz = -2\sin \lambda \, d\lambda \qquad\qquad \frac{\sqrt{4 - z^2}}{2} = \sin \lambda \qquad \Rightarrow \sqrt{4 - z^2} = 2\sin \lambda$$

$$\int \frac{dz}{\sqrt{(4 - z^2)^3}} = \int \frac{1}{(2\sin \lambda)^3}(-2\sin \lambda \, d\lambda) = -\frac{1}{4}\int \frac{d\lambda}{\sin^2 \lambda} = -\frac{1}{4}\int \cosec^2 \lambda \, d\lambda =$$

$$\frac{1}{4}\cotan \lambda + C = \frac{1}{4}\left(\frac{z}{\sqrt{4 - z^2}}\right) + C = \frac{1}{4}\left(\frac{\tan x}{\sqrt{4 - \tan^2 x}}\right) + C$$

$$\boxed{\int \frac{\sec^2 x}{(4 - \tan^2 x)^{3/2}} \, dx = \frac{1}{4}\left(\frac{\tan x}{\sqrt{4 - \tan^2 x}}\right) + C}$$

25)

$$\int \frac{e^{-x}}{(9e^{-2x} + 1)^{3/2}} \, dx = \int \frac{e^{-x}}{\sqrt{(9e^{-2x} + 1)^3}} \, dx = \qquad z = e^{-x} \qquad\qquad dz = e^{-x} \, dx$$

$$\int \frac{e^{-x}}{\sqrt{(9e^{-2x} + 1)^3}} \, dx = -\int \frac{dz}{\sqrt{(9z^2 + 1)^3}} = \qquad \tan \rho = 3z \qquad \Rightarrow \rho = \arctan 3z$$

$$z = \frac{1}{3}\tan \rho \qquad\qquad dz = \frac{1}{3}\sec^2 \rho \, d\rho \qquad\qquad \sqrt{9z^2 + 1} = \sec \rho$$

$$-\int \frac{dz}{\sqrt{(9z^2 + 1)^3}} = -\frac{1}{3}\int \left(\frac{1}{\sec^3 \rho}\right)(\sec^2 \rho \, d\rho) = -\frac{1}{3}\int \frac{d\rho}{\sec \rho} = -\frac{1}{3}\int \cos \rho \, d\rho =$$

$$-\frac{1}{3}\sin \rho + C = -\frac{1}{3}\left(\frac{3z}{\sqrt{9z^2 + 1}}\right) + C = -\frac{1}{3}\left(\frac{3e^{-x}}{\sqrt{9e^{-2x} + 1}}\right) + C$$

$$\boxed{\int \frac{e^{-x}}{(9e^{-2x} + 1)^{3/2}} \, dx = -\frac{1}{3}\left(\frac{3e^{-x}}{\sqrt{9e^{-2x} + 1}}\right) + C}$$

26)

$$\int \frac{\ln^3 x}{x\sqrt{\ln^2 x - 4}} dx = \qquad z = \ln x \qquad dz = \frac{dx}{x}$$

$$\int \frac{z^3}{\sqrt{z^2 - 4}} dz = \qquad \sin \beta = \frac{z}{2} \qquad \Rightarrow \beta = \arcsin\left(\frac{z}{2}\right)$$

$$z = 2\sin\beta \qquad dz = 2\cos\beta \, d\beta \qquad \frac{\sqrt{z^2 - 4}}{2} = \cos\beta \qquad \Rightarrow \sqrt{z^2 - 4} = 2\cos\beta$$

$$\int \frac{z^3}{\sqrt{z^2 - 4}} dz = \int \left(\frac{1}{2\cos\beta}\right)(2\sin\beta)^3 (2\cos\beta \, d\beta) = 8\int \sin^3\beta \, d\beta = 8\int \sin\beta \sin^2\beta \, d\beta$$

$$8\int \sin\beta \, (1 - \cos^2\beta) d\beta = \qquad u = \cos\beta \qquad du = -\sin\beta \, d\beta$$

$$-8\int 1 - u^2 \, du = 8\int u^2 - 1 \, du = 8\left(\frac{u^3}{3} - u\right) + C = 8u\left(\frac{u^2}{3} - 1\right) + C$$

$$8\cos\beta\left(\frac{\cos^2\beta}{3} - 1\right) + C = 4\sqrt{z^2 - 4}\left(\frac{z^2 - 4}{12} - 1\right) + C = 4\sqrt{\ln^2 x - 4}\left(\frac{\ln^2 x - 4}{12} - 1\right) + C$$

$$\boxed{\int \frac{\ln^3 x}{x\sqrt{\ln^2 x - 4}} dx = 4\sqrt{\ln^2 x - 4}\left(\frac{\ln^2 x - 4}{12} - 1\right) + C}$$

27)

$$\int \frac{x^2 + 4x - 1}{x^3 - x} dx = \int \frac{x^2 + 4x - 1}{x(x^2 - 1)} dx = \int \frac{x^2 + 4x - 1}{x(x - 1)(x + 1)} dx =$$

$$\frac{x^2 + 4x - 1}{x(x - 1)(x + 1)} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 1}$$

$$x^2 + 4x - 1 = A(x^2 - 1) + B(x^2 + x) + C(x^2 - x)$$

$$x^2 + 4x - 1 = Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx$$

$$x^2 + 4x - 1 = x^2(A + B + C) + x(B - C) - A$$

$$\begin{cases} A = 1 \\ B - C = 4 \\ A + B + C = 1 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 2 \\ C = -2 \end{cases}$$

$$\frac{x^2 + 4x - 1}{x(x-1)(x+1)} = \frac{1}{x} + \frac{2}{x-1} - \frac{2}{x+1}$$

$$\int \frac{x^2 + 4x - 1}{x(x-1)(x+1)} dx = \int \frac{dx}{x} + 2\int \frac{dx}{x-1} - 2\int \frac{dx}{x+1} =$$

$$\ln x + 2\ln(x-1) - 2\ln(x+1) + C = \ln\left(\frac{x(x-1)^2}{(x+1)^2}\right) + C$$

$$\boxed{\int \frac{x^2 + 4x - 1}{x(x-1)(x+1)} dx = \ln\left(\frac{x(x-1)^2}{(x+1)^2}\right) + C}$$

28)

$$\int \frac{3x}{2x^4 + 5x^2 + 2} dx =$$

$$2x^4 + 5x^2 + 2 = 0 \qquad \Rightarrow x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x^2 = \frac{-5 \pm \sqrt{25 - 16}}{4}$$

$$x^2 = \frac{-5 \pm 3}{4} \qquad (x^2)_1 = -\frac{1}{2} \qquad (x^2)_2 = -2$$

$$\Rightarrow 2x^4 + 5x^2 + 2 = (2x^2 + 1)(x^2 + 2)$$

$$\int \frac{3x}{2x^4 + 5x^2 + 2} dx = \int \frac{3x}{(2x^2 + 1)(x^2 + 2)} dx =$$

$$\frac{3x}{(2x^2 + 1)(x^2 + 2)} = \frac{Ax + B}{2x^2 + 1} + \frac{Cx + D}{x^2 + 2}$$

$$3x = (Ax + B)(x^2 + 2) + (Cx + D)(2x^2 + 1)$$

$$3x = Ax^3 + Bx^2 + 2Ax + 2B + 2Cx^3 + Cx + 2Dx^2 + D$$

$$3x = x^3(A + 2C) + x^2(B + 2D) + x(2A + C) + (2B + D)$$

$$\begin{cases} A + 2C = 0 \\ B + 2D = 0 \\ 2A + C = 3 \\ 2B + D = 0 \end{cases} \Rightarrow \begin{cases} A + 2C = 0 \\ 2A + C = 3 \end{cases} \wedge \begin{cases} B + 2D = 0 \\ 2B + D = 0 \end{cases}$$

$$\begin{cases} A = 2 \\ B = 0 \\ C = -1 \\ D = 0 \end{cases}$$

$$\frac{3x}{(2x^2 + 1)(x^2 + 2)} = \frac{2x}{2x^2 + 1} - \frac{x}{x^2 + 2}$$

$$\int \frac{3x}{2x^4 + 5x^2 + 2} dx = \left(\int \frac{2x}{2x^2 + 1} dx \right) - \left[\int \frac{x}{x^2 + 2} dx \right] =$$

$$\int \frac{2x}{2x^2 + 1} dx = \qquad z = 2x^2 + 1 \qquad dz = 4xdx$$

$$\frac{1}{2} \int \frac{dz}{z} = \frac{1}{2} \ln z + C = \ln(2x^2 + 1)^{1/2} + C$$

$$\int \frac{x}{x^2 + 2} dx = \qquad z = x^2 + 2 \qquad dz = 2x\, dx$$

$$\frac{1}{2} \int \frac{dz}{z} = \frac{1}{2} \ln z + C = \ln(x^2 + 2)^{1/2} + C$$

$$\int \frac{3x}{2x^4 + 5x^2 + 2} dx = \left(\int \frac{2x}{2x^2 + 1} dx \right) - \left[\int \frac{x}{x^2 + 2} dx \right] = \ln(2x^2 + 1)^{\frac{1}{2}} + \ln(x^2 + 2)^{-1/2} + C$$

$$\boxed{\int \frac{3x}{2x^4 + 5x^2 + 2} dx = \ln\left(\sqrt{\frac{2x^2 + 1}{x^2 + 2}} \right) + C}$$

29)

$$\int \sin^n x\, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n+1}{n} \int \sin^{n-2} x\, dx$$

$$\int \sin^n x\, dx = \int \sin^{n-1} x \sin x\, dx = \qquad z = \sin^{n-1} x \qquad dv = \sin x\, dx$$

$$dz = (n-1) \sin^{n-2} x \cos x\, dx \qquad v = -\cos x$$

$$\int \sin^n x\, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cos x^2\, dx$$

$$\int \sin^n x\, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x (1 - \sin^2 x) dx$$

$$\int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$[(n-1)+1] \int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx$$

$$n \int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx$$

$$\int \sin^n x \, dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{(n-1)}{n} \int \sin^{n-2} x \, dx$$

30)

$$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$$\int \sec^n x \, dx = \int \sec^{n-2} x \sec^2 x \, dx = \qquad z = \sec^{n-2} x \qquad dv = \sec^2 x \, dx$$

$$dz = (n-2) \sec^{n-3} x \sec x \tan x \, dx \qquad v = \tan x$$

$$dz = (n-2) \sec^{n-2} x \tan x \, dx$$

$$\int \sec^n x \, dx = \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x \tan^2 x \, dx$$

$$\int \sec^n x \, dx = \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x \, (\sec^2 x - 1) dx$$

$$\int \sec^n x \, dx = \sec^{n-2} x \tan x - (n-2) \int \sec^n x \, dx + (n-2) \int \sec^{n-2} x \, dx$$

$$[(n-2)+1] \int \sec^n x \, dx = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x \, dx$$

$$(n-1) \int \sec^n x \, dx = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x \, dx$$

$$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

31)

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$\int x^n e^x dx = \qquad\qquad z = x^n \qquad\qquad dz = e^x dx$$

$$dz = nx^{n-1} dx \qquad\qquad v = e^x$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

32)

$$\int \sin^m x \cos^n x \, dx = \frac{-1}{m+n} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{m+1} \int \sin^{m-2} x \cos^n x \, dx$$

$$\int \sin^m x \cos^n x \, dx = \int \sin^{n-1} x \sin x \cos^n x \, dx = \qquad z = \sin^{n-1} x \qquad dv = \sin x \cos^n x \, dx$$

$$dz = (m-1) \sin^{n-2} x \cos x \, dx$$

$$v = \int \sin x \cos^n x \, dx \;=\; u = \cos x \qquad du = -\sin x \, dx$$

$$v = \int \sin x \cos^n x \, dx \;=\; -\int u^n \, du = \frac{-1}{n+1} u^{n+1} + C = \frac{-1}{n+1} \cos^{n+1} x + C$$

$$v = \frac{-1}{n+1} \cos^{n+1} x$$

$$\int \sin^m x \cos^n x \, dx = \frac{-1}{n+1} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{n+1} \int \sin^{m-2} x \cos^{n+1} x \cos x \, dx$$

$$\int \sin^m x \cos^n x \, dx = \frac{-1}{n+1} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{n+1} \int \sin^{m-2} x \cos^{n+2} x \, dx$$

$$\int \sin^m x \cos^n x \, dx = \frac{-1}{n+1} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{n+1} \int \sin^{m-2} x \cos^n x \cos^2 x \, dx$$

$$\int \sin^m x \cos^n x \, dx = \frac{-1}{n+1} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{n+1} \int \sin^{m-2} x \cos^n x \, (1 - \sin^2 x) dx$$

$$\int \sin^m x \cos^n x \, dx = \frac{-1}{n+1} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{n+1} \left[\int \sin^{m-2} x \cos^n x \, dx - \int \sin^m x \cos^n x \, dx \right]$$

$$\left[\frac{m-1}{n+1} - 1 \right] \int \sin^m x \cos^n x \, dx = \frac{-1}{n+1} \sin^{n-1} x \cos^{n+1} x + \frac{m-1}{n+1} \int \sin^{m-2} x \cos^n x \, dx$$

$$\int \sin^m x \cos^n x \, dx = \left(\frac{-1}{n+1} \right) \left(\frac{n+1}{m-1} \right) \sin^{n-1} x \cos^{n+1} x + \left(\frac{m-1}{n+1} \right) \left(\frac{n+1}{m+n} \right) \int \sin^{m-2} x \cos^n x \, dx$$

$$\int \sin^m x \cos^n x \, dx = \left(\frac{-1}{m-1} \right) \sin^{n-1} x \cos^{n+1} x + \left(\frac{m-1}{m+n} \right) \int \sin^{m-2} x \cos^n x \, dx$$

$$\boxed{\int \sin^m x \cos^n x \, dx = \left(\frac{-1}{m-1} \right) \sin^{n-1} x \cos^{n+1} x + \left(\frac{m-1}{m+n} \right) \int \sin^{m-2} x \cos^n x \, dx}$$