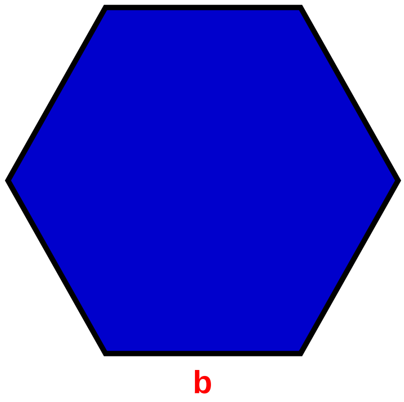
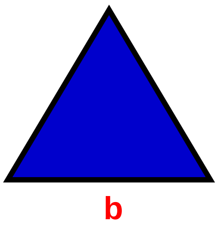
Cuerpos Platónicos truncados

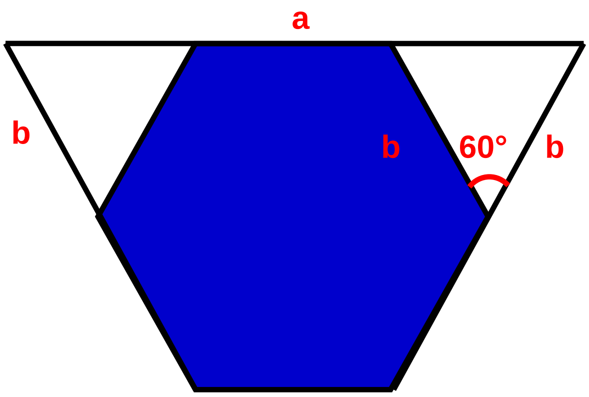
Tetraedro truncado

Construcción

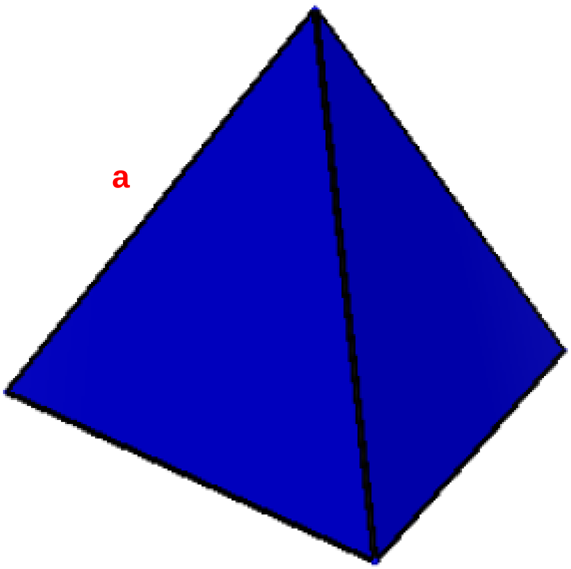
Se construyen 4 hexágonos regulares y 4 triángulos equiláteros que tengan la medida **b**.



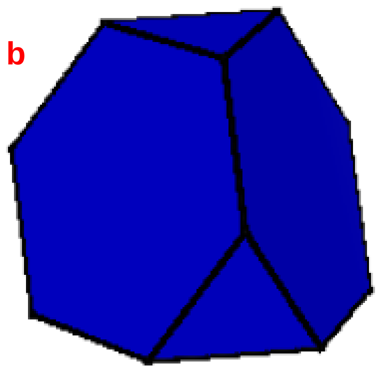




$a=3.b \leftrightarrow b=\frac{a}{3}$



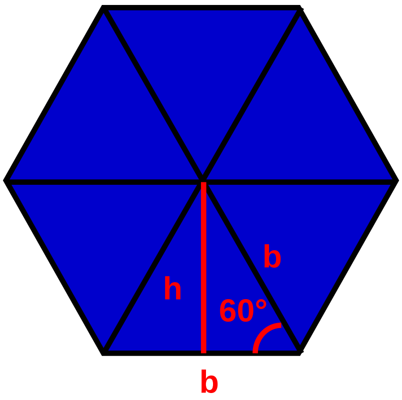
Tetraedro



Tetraedro truncado

Fórmula para el área

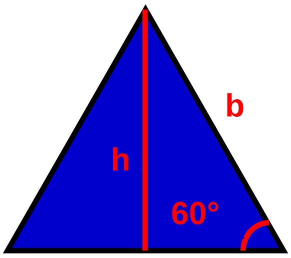
Se calcula el área de un hexágono regular y un triangulo equilátero.



$\operatorname{sen}60° = \frac{h}{b} \leftrightarrow h = b.\operatorname{sen}60° \leftrightarrow h = \frac{\sqrt{3}}{2}.b$

$A_{\text{triángulo}} = \frac{b.\sqrt{3}.b}{4} = \frac{\sqrt{3}}{4}.b^2$

$A_{\text{hexágono}} = 6.\frac{\sqrt{3}}{4}.b^2$



$\operatorname{sen}60° = \frac{h}{b} \leftrightarrow h = b.\operatorname{sen}60° \leftrightarrow h = \frac{\sqrt{3}}{2}.b$

$A_{\text{triángulo}} = \frac{b.\sqrt{3}.b}{4}$

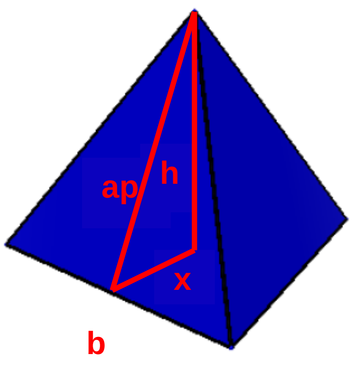
$A_{\text{triángulo}} = \frac{\sqrt{3}}{4}.b^2$

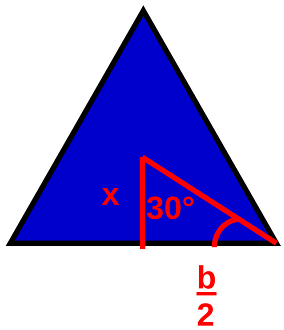
$A_{\text{tetraedro truncado}} = 4.\ 6.\frac{\sqrt{3}}{4}.b^2 + 4.\ \frac{\sqrt{3}}{4}.b^2 = 6.\sqrt{3}.b^2 + \sqrt{3}.b^2$

$\boxed{A_{\text{tetraedro truncado}} = 7.\ b^2.\sqrt{3}.}$

Fórmula para el volumen

Se calcula el volumen de la pirámide.





$\tan 30° = 2.\frac{x}{b} \leftrightarrow x = \frac{b}{2}.\tan 30° \leftrightarrow x = \frac{b}{2}.\frac{\sqrt{3}}{3} \leftrightarrow x = \frac{\sqrt{3}}{6}.b$

$ap = \frac{\sqrt{3}}{2}.b \leftrightarrow ap^2 = h^2 + x^2 \leftrightarrow h^2 = ap^2 - x^2 \leftrightarrow h^2 = (\frac{\sqrt{3}}{2}.b)^2 - (\frac{\sqrt{3}}{6}.b)^2 \leftrightarrow h^2 = \frac{3}{4}.b^2 - \frac{3}{36}.b^2 \leftrightarrow h = \sqrt{(\frac{2}{3}.b^2)} \leftrightarrow h = \frac{\sqrt{6}}{3}.b$

$V_{\text{pirámide}} = \frac{1}{3}.\frac{\sqrt{3}}{4}.b^2.\frac{\sqrt{6}}{3}.b \leftrightarrow V_{\text{pirámide}} = \frac{\sqrt{2}}{12}.b^3$

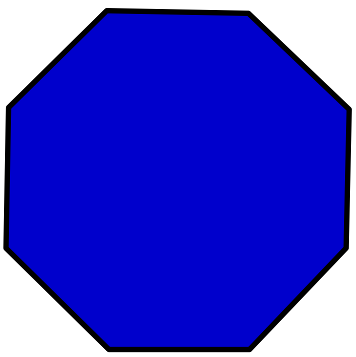
$V_{\text{tetraedro truncado}} = V_{\text{tetraedro}} - 4.V_{\text{piramide}} \leftrightarrow V_{\text{tetraedro truncado}} = \frac{\sqrt{2}}{12}.a^3 - 4.\frac{\sqrt{2}}{12}.b^3 \leftrightarrow a = 3.b$

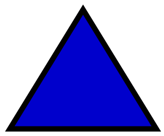
$$V_{\text{tetraedro truncado}}=27.\frac{\sqrt{2}}{12}.b^3-4.\frac{\sqrt{2}}{12}.b^3$$

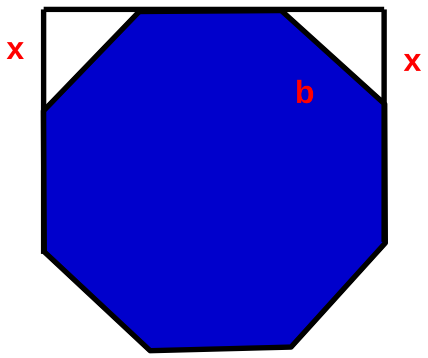
$$\boxed{V_{\text{tetraedro truncado}}=\frac{23}{12}.\ b^3.\sqrt{2}.}$$

<u>Hexaedro truncado</u>

<u>Construcción</u>
Se construyen 6 octógonos regulares y 8 triángulos equiláteros que tengan la medida **b**.



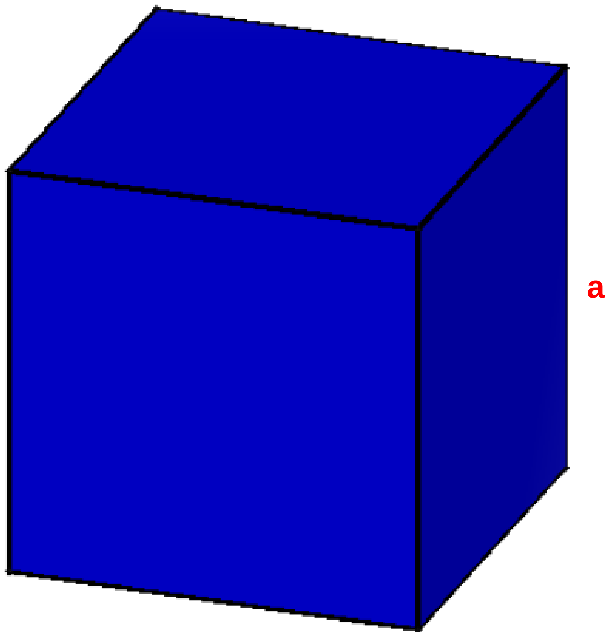




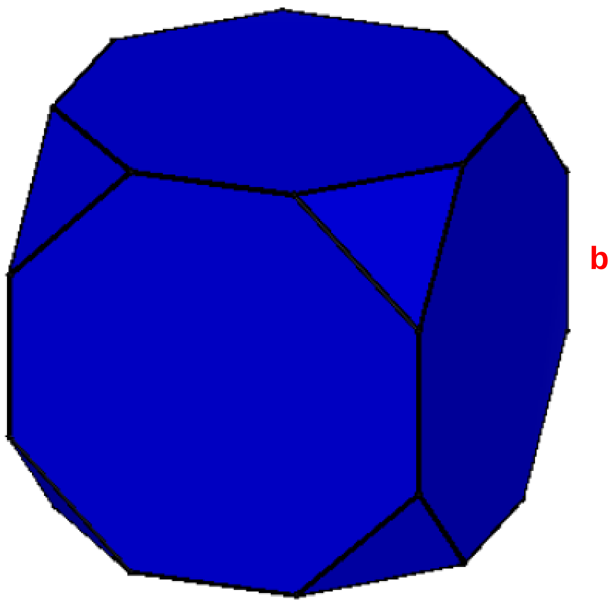
$$b^2=x^2+x^2 \leftrightarrow b^2=2.x^2 \leftrightarrow x=\frac{\sqrt{2}}{2}.b$$

$$a=b+2.x \leftrightarrow a=b+2.\frac{\sqrt{2}}{2}.b \leftrightarrow a=(1+\sqrt{2}).b$$

$$b=\frac{a}{(1+\sqrt{2})}$$



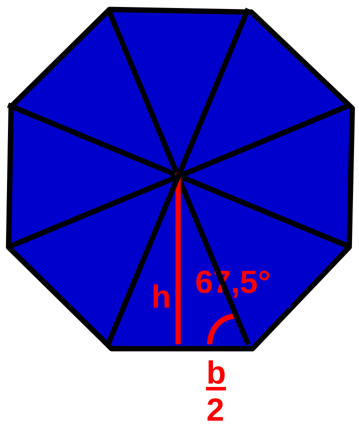
Hexaedro



Hexaedro truncado

Fórmula para el área

Se calcula el área de un octógono regular y un triangulo equilátero.

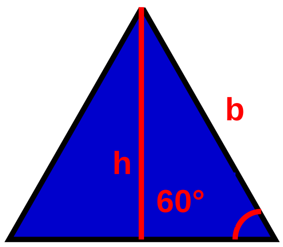


$$\tan 67{,}5° = 2.\frac{h}{b} \leftrightarrow h = \frac{b}{2}.\tan 67{,}5° \leftrightarrow h = \frac{(1+\sqrt{2})}{2}.b$$

$$A_{triángulo} = \frac{b.(1+\sqrt{2}).b}{4} = \frac{(1+\sqrt{2})}{4}.b^2$$

$$A_{octógono} = 8.\frac{(1+\sqrt{2})}{4}.b^2$$

$$A_{octógono} = 2.(1+\sqrt{2}).b^2$$



$$\text{sen}60° = \frac{h}{b} \leftrightarrow h = b.\text{sen}60° \leftrightarrow h = \frac{\sqrt{3}}{2}.b$$

$$A_{triángulo} = \frac{b.\sqrt{3}.b}{4}$$

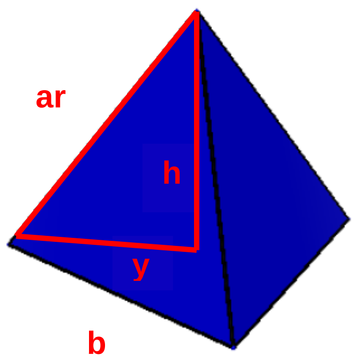
$$A_{triángulo} = \frac{\sqrt{3}}{4}.b^2$$

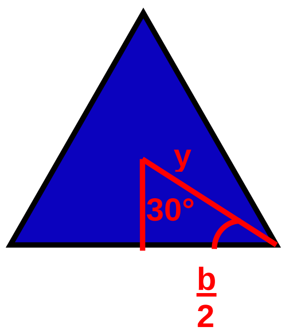
$$A_{hexaedro\ truncado} = 6.2.(1+\sqrt{2}).b^2 + 8.\frac{\sqrt{3}}{4}.b^2$$

$$\boxed{A_{hexaedro\ truncado} = 2.\ b^2.(6+6.\sqrt{2}+\sqrt{3})}$$

Fórmula para el volumen

Se calcula el volumen de la pirámide.





$$\cos 30° = \frac{b}{2.y} \leftrightarrow y = \frac{b}{2.\cos 30°} \leftrightarrow y = \frac{2.b}{2.\sqrt{3}} \leftrightarrow y = \frac{\sqrt{3}}{3}.b$$

$$ar = \frac{\sqrt{2}}{2}.b \leftrightarrow ar^2 = h^2 + y^2 \leftrightarrow h^2 = ar^2 - y^2 \leftrightarrow h^2 = \left(\frac{\sqrt{2}}{2}.b\right)^2 - \left(\frac{\sqrt{3}}{3}.b\right)^2 \leftrightarrow h^2 = \frac{2}{4}.b^2 - \frac{3}{9}.b^2 \leftrightarrow h = \sqrt{\left(\frac{1}{6}.b^2\right)} \leftrightarrow h = \frac{\sqrt{6}}{6}.b$$

$$V_{pirámide} = \frac{1}{3}.\frac{\sqrt{3}}{4}.b^2.\frac{\sqrt{6}}{6}.b \leftrightarrow V_{pirámide} = \frac{\sqrt{2}}{24}.b^3$$

$$V_{hexaedro\ truncado} = V_{hexaedro} - 8.V_{pirámide} \leftrightarrow V_{hexaedro\ truncado} = a^3 - 8.\frac{\sqrt{2}}{24}.b^3 \leftrightarrow a = (1+\sqrt{2}).b$$

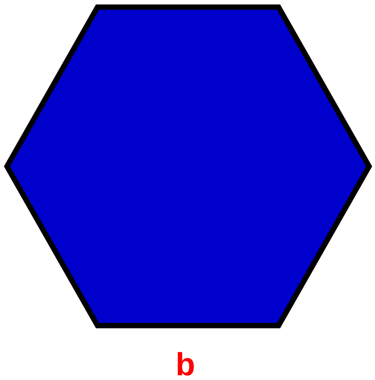
$$V_{hexaedro\ truncado} = (1+\sqrt{2})^3.b^3 - \frac{\sqrt{2}}{3}.b^3 = (1+3.\sqrt{2}+3.2+2.\sqrt{2}).b^3 - \frac{\sqrt{2}}{3}.b^3 = (7+5.\sqrt{2}).b^3 - \frac{\sqrt{2}}{3}.b^3 = \left(7+\frac{14}{3}.\sqrt{2}\right).b^3$$

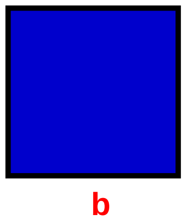
$$V_{\text{hexaedro truncado}} = \frac{7}{3} \cdot b^3 \cdot (3+2\cdot\sqrt{2})$$

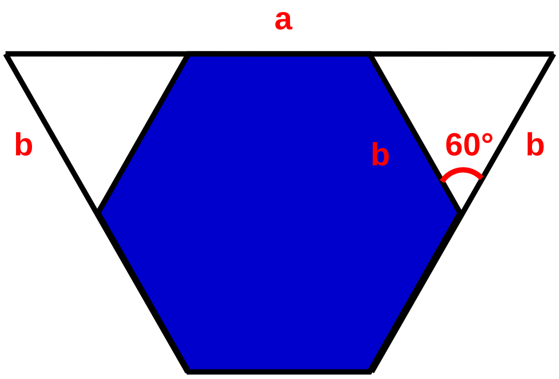
Octaedro truncado

Construcción

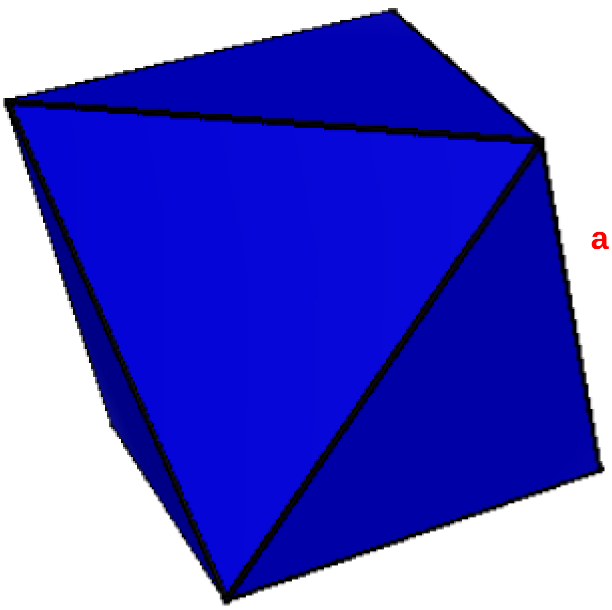
Se construyen 8 hexágonos regulares y 6 cuadrados que tengan la medida **b**.



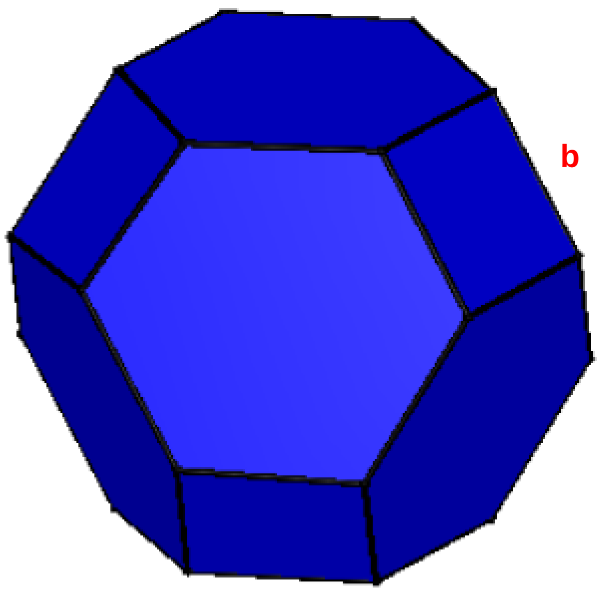




$$a = 3 \cdot b \leftrightarrow b = \frac{a}{3}$$



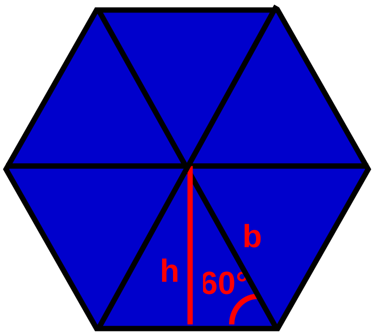
Octaedro



Octaedro truncado

Fórmula para el área

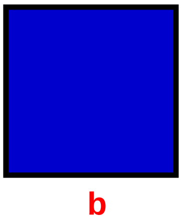
Se calcula el área de un hexágono regular y un cuadrado.



$$sen60° = \frac{h}{b} \leftrightarrow h = b.sen60° \leftrightarrow h = \frac{\sqrt{3}}{2}.b$$

$$A_{triángulo} = \frac{b.\sqrt{3}.b}{4} = \frac{\sqrt{3}}{4}.b^2$$

$$A_{hexágono} = 6.\frac{\sqrt{3}}{4}.b^2$$



$$A_{cuadrado} = b.b$$

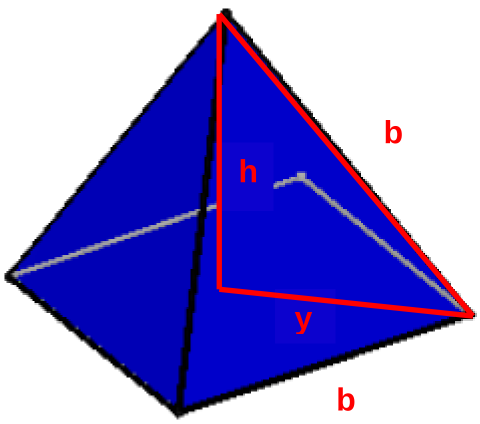
$$A_{cuadrado} = b^2$$

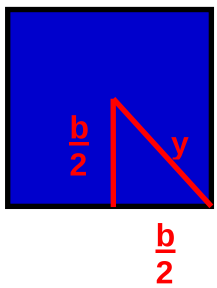
$$A_{octaedro\ truncado} = 8.6.\frac{\sqrt{3}}{4}.b^2 + 6.b^2 = 12.\sqrt{3}.b^2 + 6.b^2$$

$$\boxed{A_{octaedro\ truncado} = 6.\ b^2.(1+2.\sqrt{3})}$$

Fórmula para el volumen

Se calcula el volumen de la pirámide.





$$y^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{b}{2}\right)^2 = 2.\frac{b^2}{4} = \frac{b^2}{2} \leftrightarrow y = \frac{\sqrt{2}}{2}.b$$

$$b^2 = h^2 + y^2 \leftrightarrow h^2 = b^2 - y^2 \leftrightarrow h^2 = b^2 - \left(\frac{\sqrt{2}}{2}.b\right)^2 \leftrightarrow h^2 = b^2 - \frac{b^2}{2} \leftrightarrow h^2 = \frac{b^2}{2} \leftrightarrow h = \frac{\sqrt{2}}{2}.b$$

$$V_{pirámide} = \frac{1}{3}.b^2.\frac{\sqrt{2}}{2}.b \leftrightarrow V_{pirámide} = \frac{\sqrt{2}}{6}.b^3$$

$$V_{octaedro\ truncado} = V_{octaedro} - 6.V_{pirámide} \leftrightarrow V_{octaedro\ truncado} = \frac{\sqrt{2}}{3}.a^3 - 6.\frac{\sqrt{2}}{6}.b^3 \leftrightarrow a = 3.b$$

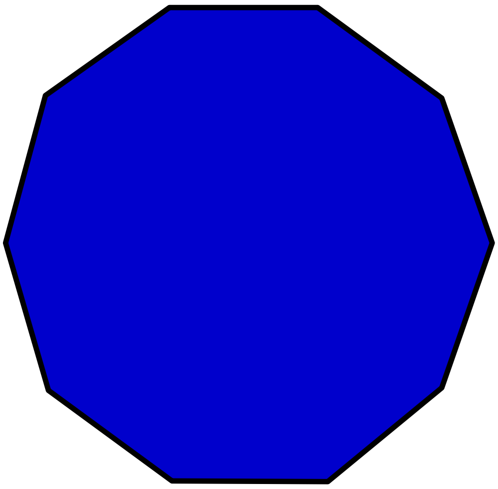
$V_{\text{octaedro truncado}}=27.\frac{\sqrt{2}}{3}.b^3-\sqrt{2}.b^3=9.\sqrt{2}.b^3-\sqrt{2}.b^3$

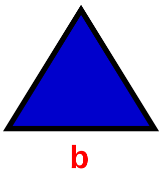
$$\boxed{V_{\text{octaedro truncado}}=8.\ b^3.\sqrt{2}}$$

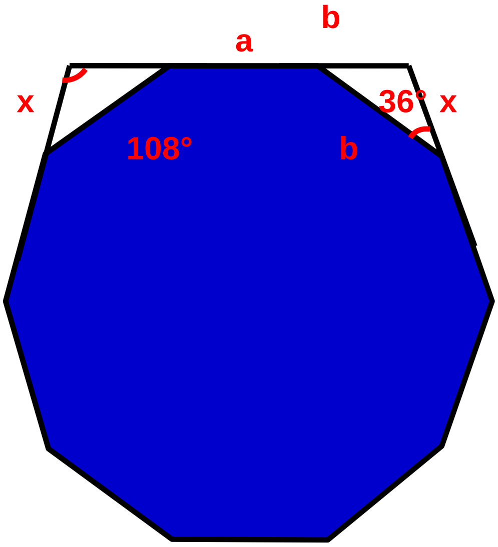
Dodecaedro truncado

Construcción

Se construyen 12 decágonos regulares y 20 triángulos equiláteros que tengan la medida **b**.







$\frac{x}{sen36°}=\frac{b}{sen108°} \leftrightarrow x=\frac{b.sen36°}{sen108°} \leftrightarrow$

$sen36°=sen2.18°=2.sen18°.cos18° \leftrightarrow$

$sen108°=sen(180°-108°)=sen72°=cos(90°-72°)=cos18° \leftrightarrow$

$x=\frac{b.2.sen18°.cos18°}{cos18°}=b.2.sen18°$

$cosn.t+i.senn.t=(cost+sent)^n \leftrightarrow n=5 \wedge t=18°$

$cos5.t+i.sen5.t=(cost+i.sent)^5$

$(cost+i.sent)^5=cos^5t+5.cos^4t.i.sent+10.cos^3t.i^2.sen^2t+10.cos^2t.i^3.sen^3t+5.cost.i^4.sen^4t+i^5.sen^5t=$

$cos^5t+5.cos^4t.i.sent-10.cos^3t.sen^2t-10.cos^2t.i.sen^3t+5.cost.sen^4t+i.sen^5t=$

$cost.(cos^4t-10.cos^2t.sen^2t+5.sen^4t)+i.sent.(5.cos^4t-10.cos^2t.sen^2t+sen^4t)=$

$cost.[cos^4t-10.cos^2t.(1-cos^2t)+5.(1-cos^2t)^2]+i.sent.[5.(1-sen^2t)^2-10.(1-sen^2t).sen^2t+sen^4t]=$

$cost.(cos^4t-10.cos^2t+10.cos^4t+5-10.cos^2t+5.cos^4t)+i.sent.(5-10.sen^2t+5.sen^4t-$

$10.sen^2t+10.sen^4t+sen^4t)= cost.(16.cos^4t-20.cos^2t+5)+i.sent.(16.sen^4t-20.sen^2t+5)=$

$16.cos^5t-20.cos^3t+5.cost+i.(16.sen^5t-20.sen^3t+5.sent)=cos5.t+i.sen5.t \leftrightarrow$

$cos5.t=16.cos^5t-20.cos^3t+5.cost \wedge sen5.t=16.sen^5t-20.sen^3t+5.sent \leftrightarrow t=18°,\ cost=x \wedge sent=y$

$\cos90°=16.x^5-20.x^3+5.x=0 \wedge \text{sen}90°=16.y^5-20.y^3+5.y=1$

$16.x^4-20.x^2+5=0 \wedge 16.y^5-20.y^3+5.y-1=0 \leftrightarrow$

$(16.y^5-20.y^3+5.y-1):(y-1)=16.y^4+16.y^3-4.y^2-4.y+1 \leftrightarrow 16.y^4+16.y^3-4.y^2-4.y+1=0$

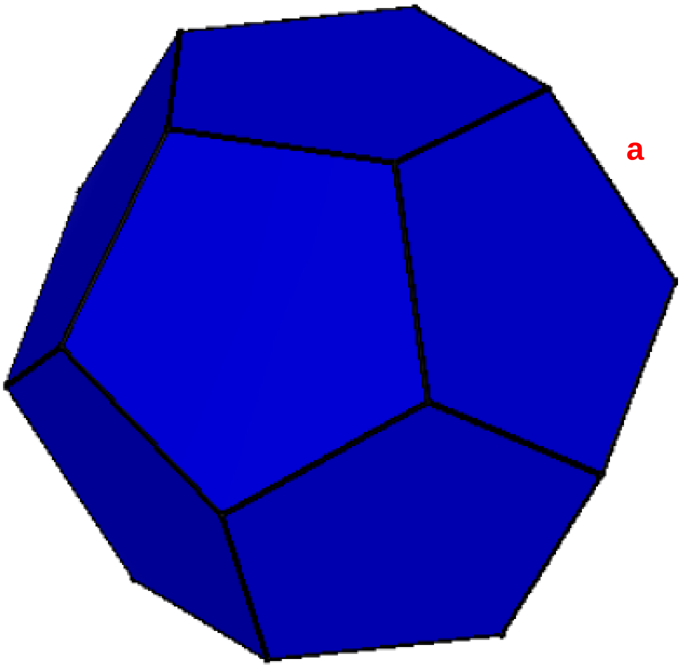
$16.y^4+16.y^3+4.y^2-8.y^2-4.y+1=(4.y^2)^2+2.4.y^2.2.y+(2.y)^2-2.(4.y^2+2.y)+1=(4.y^2+2.y)^2-2.(4.y^2+2.y).1+1^2=$

$[(4.y^2+2.y)-1]^2=0 \leftrightarrow 4.y^2+2.y-1=0 \leftrightarrow y_{1-2-3-4}=\frac{-2\pm\sqrt{(4+16)}}{8}=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm2.\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4} \leftrightarrow$

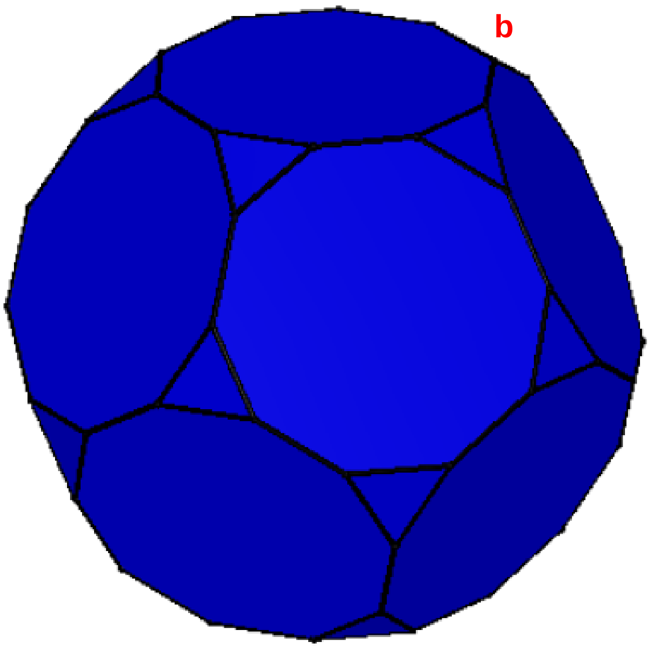
$\text{sen}18°=\frac{-1+\sqrt{5}}{4}$

$x=b.2.(\frac{-1+\sqrt{5}}{4})=\frac{b}{2}.(-1+\sqrt{5})$

$a=b+2.x=b+2.\frac{b}{2}.(-1+\sqrt{5})=b.(1-1+\sqrt{5})=b.\sqrt{5} \leftrightarrow b=\frac{a}{\sqrt{5}}=a.\frac{\sqrt{5}}{5}$



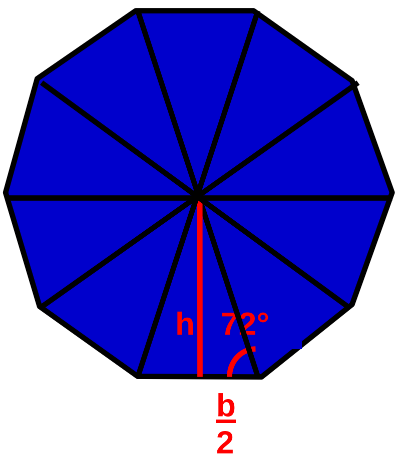
Dodecaedro



Dodecaedro truncado

Fórmula para el área

Se calcula el área de un decágono regular y un triangulo equilátero.



$\tan72°=2.\frac{h}{b} \leftrightarrow h=\frac{b}{2}.\tan72°=\frac{b.\text{sen}72°}{2.\cos72°}$

$\cos 5.t=16.\cos^5 t-20.\cos^3 t+5.\cos t \wedge \operatorname{sen} 5.t=16.\operatorname{sen}^5 t-20.\operatorname{sen}^3 t+5.\operatorname{sen} t \leftrightarrow t=72°,\ \cos t=x \wedge \operatorname{sen} t=y$

$\cos 360°=16.x^5-20.x^3+5.x=1 \wedge \operatorname{sen} 360°=16.y^5-20.y^3+5.y=0$

$16.x^5-20.x^3+5.x-1=0 \wedge 16.y^4-20.y^2+5=0 \leftrightarrow$

$16.y^4-20.y^2+5=0 \leftrightarrow y_{1-2-3-4}=\pm\sqrt{\left[\frac{20\pm\sqrt{(400-320)}}{32}\right]}=\pm\sqrt{\left(\frac{20\pm\sqrt{80}}{32}\right)}=\pm\sqrt{\left(\frac{20\pm 4.\sqrt{5}}{32}\right)}=\pm\sqrt{\left(\frac{10\pm 2.\sqrt{5}}{16}\right)}=$

$\pm\frac{\sqrt{(10\pm 2.\sqrt{5})}}{4} \leftrightarrow \operatorname{sen} 72°=\frac{\sqrt{(10+2.\sqrt{5})}}{4}$

$(16.x^5-20.x^3+5.x-1):(x-1)=16.x^4+16.x^3-4.x^2-4.x+1 \leftrightarrow 16.x^4+16.x^3-4.x^2-4.x+1=0$

$16.x^4+16.x^3+4.x^2-8.x^2-4.x+1=(4.x^2)^2+2.4.x^2.2.x+(2.x)^2-2.(4.x^2+2.x)+1=(4.x^2+2.x)^2-2.(4.x^2+2.x).1+1^2=$

$[(4.x^2+2.x)-1]^2=0 \leftrightarrow 4.x^2+2.x-1=0 \leftrightarrow x_{1-2-3-4}=\frac{-2\pm\sqrt{(4+16)}}{8}=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm 2.\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4} \leftrightarrow$

$\cos 72°=\frac{-1+\sqrt{5}}{4} \leftrightarrow$

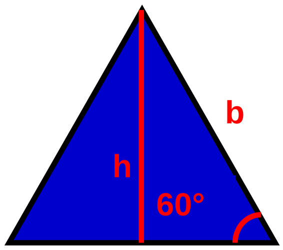
$h=\frac{b.\sqrt{(10+2.\sqrt{5})}.4}{2.(-1+\sqrt{5}).4}=\frac{b.(-2-2.\sqrt{5}).\sqrt{(10+2.\sqrt{5})}}{4-20}=\frac{b.\sqrt{[(2+2.\sqrt{5})^2.(10+2.\sqrt{5})]}}{16}=\frac{b.\sqrt{[(4+8.\sqrt{5}+20).(10+2.\sqrt{5})]}}{16}=$

$\frac{b.\sqrt{[(24+8.\sqrt{5}).(10+2.\sqrt{5})]}}{16}=\frac{b.\sqrt{(240+48.\sqrt{5}+80.\sqrt{5}+80)}}{16}=\frac{b.\sqrt{(320+128.\sqrt{5})}}{16}=\frac{b.\sqrt{[64.(5+2.\sqrt{5})]}}{16}=$

$=\frac{b.8.\sqrt{(5+2.\sqrt{5})}}{16}=\frac{b}{2}.\sqrt{(5+2.\sqrt{5})}$

$A_{triángulo}=\frac{b.b.\sqrt{(5+2.\sqrt{5})}}{2.2}=\frac{b^2}{4}.\sqrt{(5+2.\sqrt{5})}$

$A_{decágono}=10.\frac{b^2}{4}.\sqrt{(5+2.\sqrt{5})}=\frac{5.b^2}{2}.\sqrt{(5+2.\sqrt{5})}$



$\operatorname{sen} 60°=\frac{h}{b} \leftrightarrow h=b.\operatorname{sen} 60° \leftrightarrow h=\frac{\sqrt{3}}{2}.b$

$A_{triángulo}=\frac{b.\sqrt{3}.b}{4}$

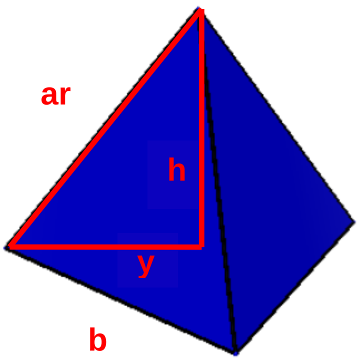
$A_{triángulo}=\frac{\sqrt{3}.b^2}{4}$

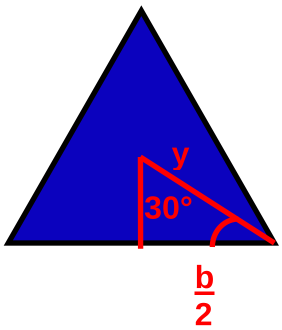
$A_{dodecaedro\ truncado}=12.\frac{5.b^2}{2}.\sqrt{(5+2.\sqrt{5})}+20.\frac{\sqrt{3}.b^2}{4}$

$\boxed{A_{dodecaedro\ truncado}=5.b^2.[6.\sqrt{(5+2.\sqrt{5})}+\sqrt{3}]}$

Fórmula para el volumen

Se calcula el volumen de la pirámide.





$\cos 30° = \frac{b}{2.y} \leftrightarrow y = \frac{b}{2.\cos 30°} \leftrightarrow y = \frac{2.b}{2.\sqrt{3}} \leftrightarrow y = \frac{\sqrt{3}.b}{3}$

$ar = \frac{b}{2}.(-1+\sqrt{5}) \leftrightarrow ar^2 = h^2 + y^2 \leftrightarrow h^2 = ar^2 - y^2 \leftrightarrow h^2 = \left[\frac{b}{2}.(-1+\sqrt{5})\right]^2 - \left(\frac{\sqrt{3}}{3}.b\right)^2 = \frac{b^2}{4}.(-1+\sqrt{5})^2 - \frac{b^2}{3} = \frac{b^2}{4}.(1-2.\sqrt{5}+5) - \frac{b^2}{3} =$

$\frac{b^2}{4}.(6-2.\sqrt{5}) - \frac{b^2}{3} = \frac{b^2}{2}.(3-\sqrt{5}) - \frac{b^2}{3} = b^2.\left(\frac{7}{6}-\sqrt{5}\right) = \frac{b^2}{6}.(7-6.\sqrt{5}) \leftrightarrow h = b.\frac{\sqrt{6}}{6}.\sqrt{(7-6.\sqrt{5})}$

$V_{pirámide} = \frac{1}{3}.\frac{\sqrt{3}}{4}.b^2.b.\frac{\sqrt{6}}{6}.\sqrt{(7-6.\sqrt{5})}. \leftrightarrow V_{pirámide} = b^3.\frac{\sqrt{2}}{24}.\sqrt{(7-6.\sqrt{5})}$

$V_{dodecaedro\ truncado} = V_{dodecaedro} - 20.V_{pirámide} \leftrightarrow V_{dodecaedro\ truncado} = \frac{5}{2}.a^3.\sqrt{(47+21.\sqrt{5})} - 20.b^3.\frac{\sqrt{2}}{24}.\sqrt{(7-6.\sqrt{5})} \leftrightarrow$

$a = b.\sqrt{5}$

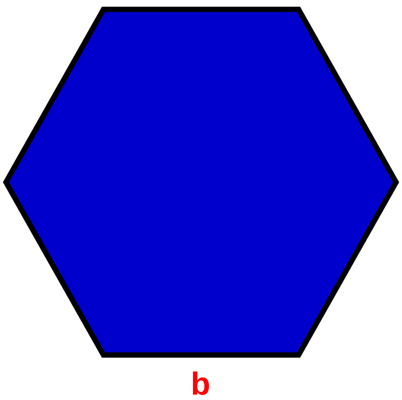
$V_{dodecaedro\ truncado} = \frac{5}{2}.(b.\sqrt{5})^3.\sqrt{(47+21.\sqrt{5})} - \frac{5}{6}.b^3.\sqrt{2}.\sqrt{(7-6.\sqrt{5})} = \frac{25}{2}.b^3.\sqrt{5}.\sqrt{(47+21.\sqrt{5})} - \frac{5}{6}.b^3.\sqrt{2}.\sqrt{(7-6.\sqrt{5})} =$

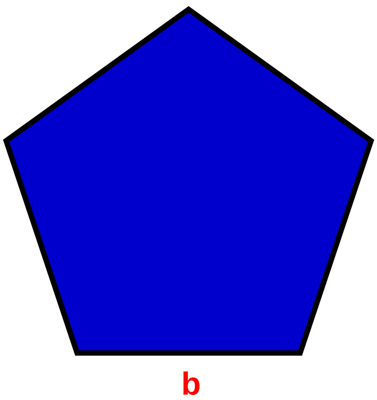
$$\boxed{V_{dodecaedro\ truncado} = \frac{5}{6}.b^3.[15.\sqrt{5}.\sqrt{(47+21.\sqrt{5})} - \sqrt{2}.\sqrt{(7-6.\sqrt{5})}]}$$

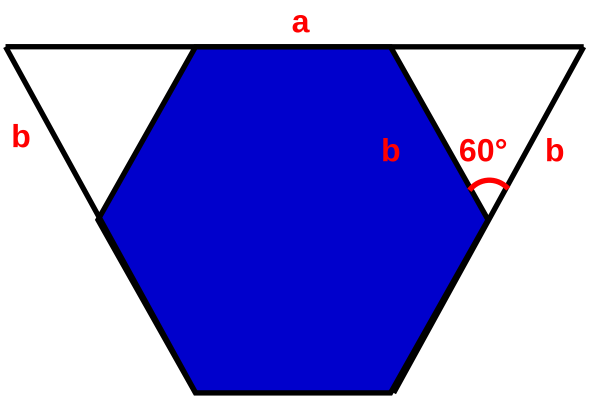
Icosaedro truncado

Construcción

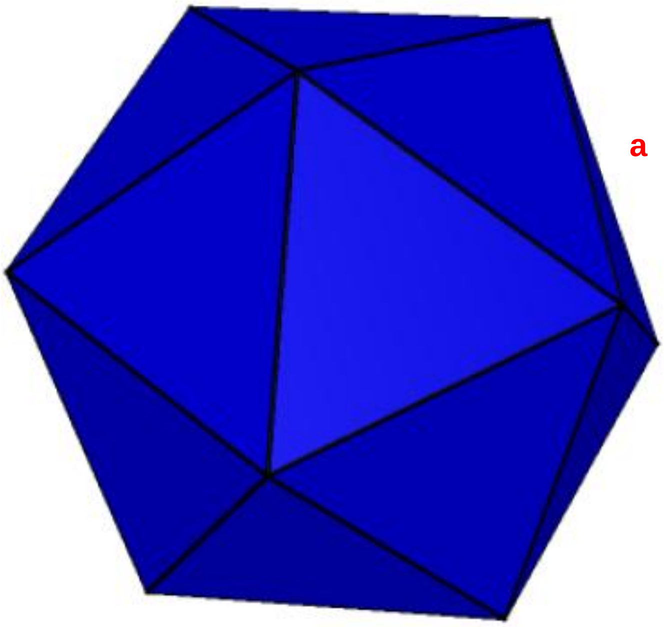
Se construyen 20 hexágonos regulares y 12 pentágonos regulares que tengan la medida **b**.



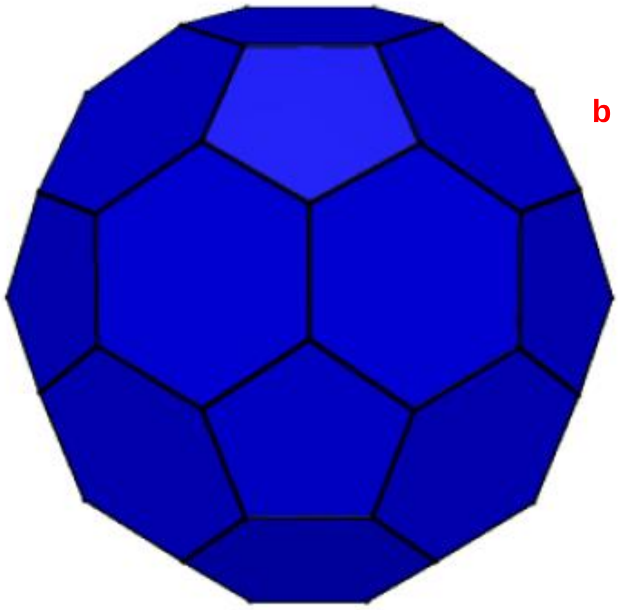




$a = 3.b \leftrightarrow b = \frac{a}{3}$

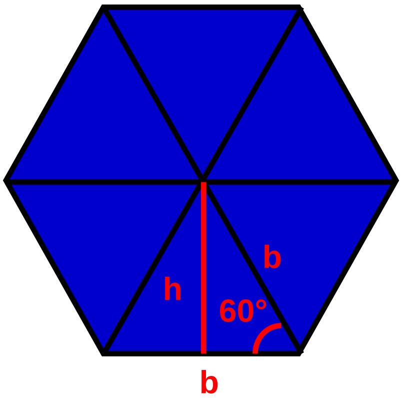


Icosaedro



Icosaedro truncado

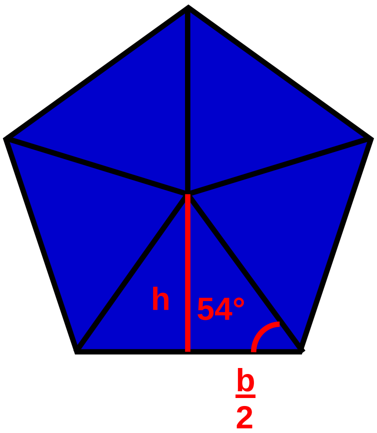
<u>Fórmula para el área</u>
Se calcula el área de un hexágono regular y un pentágono regular.



$\mathrm{sen}60° = \frac{h}{b} \leftrightarrow h = b.\mathrm{sen}60° \leftrightarrow h = \frac{\sqrt{3}.b}{2}$

$A_{triángulo} = \frac{b.\sqrt{3}.b}{2.2} = \frac{\sqrt{3}.b^2}{4}$

$A_{hexágono} = 6.\frac{\sqrt{3}.b^2}{4} = \frac{3.\sqrt{3}.b^2}{2}$



$\tan 54° = 2.\frac{h}{b} \leftrightarrow h = \frac{b}{2}.\tan 54° = \frac{b.\mathrm{sen}54°}{2.\cos 54°}$

$\cos 5.t = 16.\cos^5 t - 20.\cos^3 t + 5.\cos t \wedge \mathrm{sen}5.t = 16.\mathrm{sen}^5 t - 20.\mathrm{sen}^3 t + 5.\mathrm{sen}\,t \leftrightarrow t = 54°$, $\cos t = x \wedge \mathrm{sen}\,t = y$

$\cos 270° = 16.x^5 - 20.x^3 + 5.x = 0 \wedge \mathrm{sen}270° = 16.y^5 - 20.y^3 + 5.y = -1$

$$16.x^4-20.x^2+5=0 \wedge 16.y^5-20.y^3+5.y+1=0 \leftrightarrow$$

$$(16.y^5-20.y^3+5.y+1):(y+1)=16.y^4-16.y^3-4.y^2+4.y+1 \leftrightarrow 16.y^4-16.y^3-4.y^2+4.y+1=0$$

$$16.y^4-16.y^3+4.y^2-8.y^2+4.y+1=(4.y^2)^2-2.4.y^2.2.y+(2.y)^2-2.(4.y^2-2.y)+1=(4.y^2-2.y)^2-2.(4.y^2-2.y).1+1^2=$$

$$[(4.y^2-2.y)-1]^2=0 \leftrightarrow 4.y^2-2.y-1=0 \leftrightarrow y_{1-2-3-4}=\frac{2\pm\sqrt{(4+16)}}{8}=\frac{2\pm\sqrt{20}}{8}=\frac{2\pm2.\sqrt{5}}{8}=\frac{1\pm\sqrt{5}}{4} \leftrightarrow \text{sen}54°=\frac{1+\sqrt{5}}{4}$$

$$16.x^4-20.x^2+5=0 \leftrightarrow x_{1-2-3-4}=\pm\sqrt{\frac{20\pm\sqrt{(400-320)}}{32}}=\pm\sqrt{\frac{20\pm\sqrt{80}}{32}}=\pm\sqrt{\frac{20\pm4.\sqrt{5}}{32}}=\pm\sqrt{\frac{10\pm2.\sqrt{5}}{16}}=$$

$$\frac{\pm\sqrt{(10\pm2.\sqrt{5})}}{4} \leftrightarrow \cos54°=\frac{\sqrt{(10-2.\sqrt{5})}}{4}$$

$$h=\frac{b.(1+\sqrt{5}).4}{2.\sqrt{(10-2.\sqrt{5})}.4}=\frac{b.(1+\sqrt{5}).\sqrt{(10+2.\sqrt{5})}}{2.\sqrt{80}}=\frac{b.\sqrt{[(1+\sqrt{5})^2.(10+2.\sqrt{5})]}}{8.\sqrt{5}}=\frac{b.\sqrt{5}.\sqrt{[(1+2.\sqrt{5}+5).(10+2.\sqrt{5})]}}{40}=$$

$$\frac{b.\sqrt{5}.\sqrt{[(6+2.\sqrt{5}).(10+2.\sqrt{5})]}}{40}=\frac{b.\sqrt{5}.\sqrt{(60+12.\sqrt{5}+20.\sqrt{5}+20)}}{40}=\frac{b.\sqrt{5}.\sqrt{(80+32.\sqrt{5})}}{40}=\frac{b.\sqrt{5}.\sqrt{[16.(5+2.\sqrt{5})]}}{40}=$$

$$\frac{b.\sqrt{5}.4.\sqrt{(5+2.\sqrt{5})}}{40}=\frac{b.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}}{10}$$

$$A_{\text{triángulo}}=\frac{b.b.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}}{2.10}=\frac{b^2.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}}{20}$$

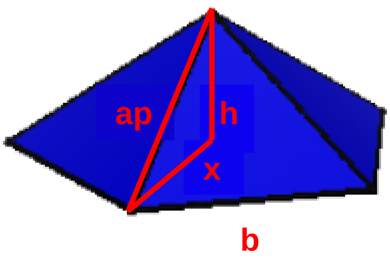
$$A_{\text{pentágono}}=5.\frac{b^2.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}}{20}=\frac{b^2}{4}.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}$$

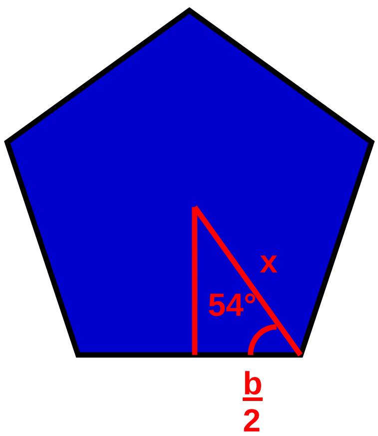
$$A_{\text{icosaedro truncado}}=20.\frac{3}{2}.\sqrt{3}.b^2+\frac{12}{4}.b^2.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}$$

$$\boxed{A_{\text{icosaedro truncado}}=3.b^2.[10.\sqrt{3}+\sqrt{5}.\sqrt{(5+2.\sqrt{5})}]}$$

Fórmula para el volumen

Se calcula el volumen de la pirámide.





$$\cos54°=\frac{b}{2.x} \leftrightarrow x=\frac{b}{2.\cos54°} \leftrightarrow x=\frac{b.4}{2.\sqrt{(10-2.\sqrt{5})}}=\frac{b.2.\sqrt{(10+2.\sqrt{5})}}{\sqrt{80}}=\frac{b.2.\sqrt{(10+2.\sqrt{5})}}{4.\sqrt{5}}=\frac{b.\sqrt{5}}{10}.\sqrt{(10+2.\sqrt{5})}$$

$$ap=b \leftrightarrow ap^2=h^2+x^2 \leftrightarrow h^2=ap^2-x^2 \leftrightarrow h^2=b^2-[\frac{b.\sqrt{5}}{10}.\sqrt{(10+2.\sqrt{5})}]^2=b^2-\frac{b^2}{20}.(10+2.\sqrt{5})=b^2-\frac{b^2}{20}.2.(5+\sqrt{5})=$$

$$b^2.(1-\frac{1}{2}-\frac{\sqrt{5}}{10})=\frac{b^2}{10}.(5-\sqrt{5}) \leftrightarrow h=\frac{b}{10}.\sqrt{10}.\sqrt{(5-\sqrt{5})}$$

$$V_{\text{pirámide}}=\frac{1}{3}.\frac{b^2}{4}.\sqrt{5}.\sqrt{(5+2.\sqrt{5})}.\frac{b}{10}.\sqrt{10}.\sqrt{(5-\sqrt{5})}=\frac{b^3}{24}.\sqrt{2}.\sqrt{[(5+2.\sqrt{5}).(5-\sqrt{5})]}=\frac{b^3}{24}.\sqrt{2}.\sqrt{(25-5.\sqrt{5}+10.\sqrt{5}-10)}=$$

$$\frac{b^3}{24}.\sqrt{2}.\sqrt{(15+5.\sqrt{5})}=\frac{b^3}{24}.\sqrt{10}.\sqrt{(3+\sqrt{5})}$$

$V_{icosaedro\ truncado}=V_{icosaedro}-12.V_{piramide} \leftrightarrow V_{icosaedro\ truncado}=\frac{5.a^3}{12}.\sqrt{2}.\sqrt{(7+3.\sqrt{5})}-12.\frac{b^3}{24}.\sqrt{10}.\sqrt{(3+\sqrt{5})} \leftrightarrow$

$a=3.b \leftrightarrow V_{icosaedro\ truncado}=\frac{5.27.b^3}{12}.\sqrt{2}.\sqrt{(7+3.\sqrt{5})}-\frac{b^3}{12}.\sqrt{10}.\sqrt{(3+\sqrt{5})}$

$$\boxed{V_{icosaedro\ truncado}=\frac{b^3}{12}.[135.\sqrt{2}.\sqrt{(7+3.\sqrt{5})}-\sqrt{10}.\sqrt{(3+\sqrt{5})}]}$$

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