

INTEGRALES POR SUSTITUCIONES TRIGONOMETRICAS

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RESUMEN METODO DE SUSTITUCIÓN TRIGONOMETRICA

Un buen numero de integrales que contienen polinomios de segundo grado, se pueden transformar a integrales directas o inmediatas si se utilizan sustituciones de variables que contienen funciones trigonometricas que transforman la expresión en una identidad trigonométrica

$$a^2 + x^2 \rightarrow x = a \operatorname{tg} z \rightarrow x^2 = a^2 \operatorname{tg}^2 z$$

Reemplazando

$$\begin{aligned} a^2 + x^2 &= a^2 + (a^2 \operatorname{tg}^2 z) \\ a^2 + x^2 &= \{a^2 + a^2 \operatorname{tg}^2 z\} \\ a^2 + x^2 &= \{a^2 (1 + \operatorname{tg}^2 z)\} \\ a^2 + x^2 &= a^2 (\sec^2 z) \end{aligned}$$

$$a^2 - x^2 \rightarrow x = a \operatorname{sen} z \rightarrow x^2 = a^2 \operatorname{sen}^2 z$$

Reemplazando

$$\begin{aligned} a^2 - x^2 &= a^2 - (a^2 \operatorname{sen}^2 z) \\ a^2 - x^2 &= \{a^2 - a^2 \operatorname{sen}^2 z\} \\ a^2 - x^2 &= \{a^2 (1 - \operatorname{sen}^2 z)\} \\ a^2 - x^2 &= a^2 (\cos^2 z) \end{aligned}$$

$$x^2 - a^2 \rightarrow x = a \operatorname{sec} z \rightarrow x^2 = a^2 \operatorname{sec}^2 z$$

Reemplazando

$$\begin{aligned} x^2 - a^2 &= (a^2 \operatorname{sec}^2 z) - a^2 \\ x^2 - a^2 &= \{a^2 \operatorname{sec}^2 z - a^2\} \\ x^2 - a^2 &= \{a^2 (\sec^2 z - 1)\} \\ x^2 - a^2 &= a^2 (\operatorname{tg}^2 z) \end{aligned}$$

TABLA DE INTEGRALES

$$\int du = u + c$$

$\int a \, du = a u + c$ donde a es una constante

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c \Rightarrow n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + c$$

$$\int a^u \, du = \frac{a^u}{\ln a} + c \text{ donde } a > 0 \quad y \quad a \neq 1$$

$$\int e^u \, du = e^u + c$$

$$\int \sin u \, du = -\cos u + c$$

$$\int \cos u \, du = \sin u + c$$

$$\int \sec^2 u \, du = \tan u + c$$

$$\int \csc^2 u \, du = -\cot u + c$$

$$\int \sec u \tan u \, du = \sec u + c$$

$$\int \csc u \cot u \, du = -\csc u + c$$

$$\int \tan u \, du = \ln|\sec u| + c$$

$$\int \cot u \, du = \ln|\sin u| + c$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + c$$

$$\int \csc u \, du = \ln|\csc u - \cot u| + c = \ln\left|\tan\frac{1}{2}u\right| + c$$

Las siguientes integrales se pueden usar para resolver en forma directa, además se pueden demostrar

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arc tg} \left(\frac{x}{a} \right) + C$$

$$\int \frac{dy}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{arc sen} \left(\frac{x}{a} \right) + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arc sec} \left(\frac{x}{a} \right) + C \quad \text{donde } a > 0$$

$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln \left| \sqrt{a^2 + x^2} + x \right| + C_1$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \ln \left| \sqrt{x^2 - a^2} + x \right| + C_1$$

$$\int \sqrt{a^2 - x^2} = \frac{a^2}{2} \operatorname{arc sen} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + C$$

$$\int \frac{dx}{x \sqrt{x^2 + a^2}} = \frac{1}{a} \ln \left| \frac{\sqrt{x^2 + a^2} - a}{x} \right| + C$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln \left| \sqrt{x^2 + a^2} + x \right| + C_1$$

$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = - \frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = - \frac{\sqrt{a^2 - x^2}}{x} - \operatorname{arc sen} \frac{x}{a} + C$$

$$\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}} = \frac{x}{\sqrt{a^2 - x^2}} - \arcsen \frac{x}{a} + c$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arc tg} \left(\frac{x}{a} \right) + c$$

$$\int \frac{dx}{a^2 + x^2}$$

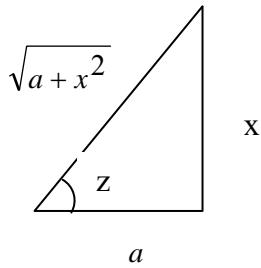
$$\int \frac{dx}{a^2 + x^2} = \int \frac{a \sec^2 z dz}{a^2 \sec^2 z}$$

$$\int \frac{a \sec^2 z dz}{a^2 \sec^2 z} = \int \frac{1}{a} dz$$

$$\int \frac{1}{a} dz = \frac{1}{a} \int dz = \frac{1}{a}(z) + c$$

Reemplazando

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}(z) + c$$



$$a^2 + x^2 \Rightarrow x = a \operatorname{tg} z$$

$$x = a \operatorname{tg} z \\ x^2 = a^2 \operatorname{tg}^2 z$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow dx = a \sec^2 z dz$$

$$a^2 + x^2 = a^2 + a^2 \operatorname{tg}^2 z$$

$$a^2 + x^2 = a^2 (1 + \operatorname{tg}^2 z)$$

$$a^2 + x^2 = a^2 (\sec^2 z)$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{a} \\ z = \operatorname{arc tg} \left(\frac{x}{a} \right)$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{a^2 - x^2} =$$

$$\int \frac{dx}{a^2 - x^2} = \int \frac{a \cos z dz}{a^2 \cos^2 z}$$

Simplificando

$$\int \frac{dx}{a^2 - x^2} = \int \frac{dz}{a \cos z} = \frac{1}{a} \int \frac{1}{\cos z} dz = \frac{1}{a} \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \tan z| + c$$

Reemplazando

$$\frac{1}{a} \int \sec z dz = \frac{1}{a} \ln |\sec z + \tan z| + c$$

$$\frac{1}{a} \int \sec z dz = \frac{1}{a} \ln \left| \frac{a}{\sqrt{a^2 - x^2}} + \frac{x}{\sqrt{a^2 - x^2}} \right| + c$$

$$\frac{1}{a} \int \sec z dz = \frac{1}{a} \ln \left| \frac{a+x}{\sqrt{a^2 - x^2}} \right| + c$$

$$\frac{1}{a} \int \sec z dz = \frac{1}{a(2)} \ln \left| \frac{(a+x)^2}{(\sqrt{a^2 - x^2})^2} \right| + c = \frac{1}{2a} \ln \left| \frac{(a+x)(a+x)}{a^2 - x^2} \right| + c$$

Cancelando términos semejantes

$$\frac{1}{2a} \ln \left| \frac{(a+x)(a+x)}{(a-x)(a+x)} \right| + c = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

$$a^2 - x^2 \Rightarrow x = a \sin z$$

$$(x)^2 = a^2 \sin^2 z dz$$

Si $x = a \sin z \rightarrow dx = a \cos z dz$

$$a^2 - (x)^2 = a^2 - a^2 \sin^2 z$$

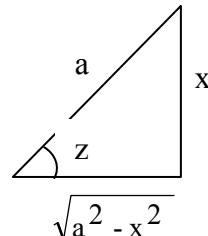
$$a^2 - x^2 = a^2 (1 - \sin^2 z)$$

$$a^2 - x^2 = a^2 (\cos^2 z)$$

$$\text{si } x = a \sin z \Rightarrow \sin z = \frac{x}{a}$$

$$\text{si } \cos z = \frac{\sqrt{a^2 - x^2}}{a} \Rightarrow \sec z = \frac{a}{\sqrt{a^2 - x^2}}$$

$$\tan z = \frac{x}{\sqrt{a^2 - x^2}}$$



$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{x^2 - a^2} =$$

$$\int \frac{dx}{x^2 - a^2} = \int \frac{a \sec z \operatorname{tg} z dz}{a^2 \operatorname{tg}^2 z}$$

$$\int \frac{\sec z dz}{a \operatorname{tg} z} = \frac{1}{a} \int \frac{1}{\cos z} \frac{1}{\operatorname{tg} z} dz$$

$$\frac{1}{a} \int \frac{1}{\cos z} \operatorname{ctg} z dz = \frac{1}{a} \int \frac{1}{\cos z} \frac{\cos z}{\sin z} dz$$

$$\frac{1}{a} \int \frac{1}{\sin z} dz = \frac{1}{a} \int \csc z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \operatorname{ctg} z| + c$$

$$\frac{1}{a} \int \csc z dz = \frac{1}{a} \ln |\csc z - \operatorname{ctg} z| + c$$

Reemplazando

$$\frac{1}{a} \ln |\csc z - \operatorname{ctg} z| + c = \frac{1}{a} \ln \left| \frac{x}{\sqrt{x^2 - a^2}} - \frac{a}{\sqrt{x^2 - a^2}} \right| + c$$

$$\begin{aligned} \frac{1}{a} \ln \left| \frac{x-a}{\sqrt{x^2 - a^2}} \right| + c &= \frac{1}{2a} \ln \left| \frac{(x-a)^2}{(\sqrt{x^2 - a^2})^2} \right| + c \\ \frac{1}{2a} \ln \left| \frac{(x-a)(x-a)}{x^2 - a^2} \right| + c &= \frac{1}{2a} \ln \left| \frac{(x-a)(x-a)}{(x-a)(x+a)} \right| + c \end{aligned}$$

Cancelando términos semejantes

$$\frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\begin{aligned} x^2 - a^2 &\Rightarrow x = a \sec z \\ x^2 = a^2 \sec^2 z & \end{aligned}$$

Si $x = a \sec z \rightarrow dx = a \sec z \operatorname{tg} z dz$

$$x^2 - a^2 = a^2 \sec^2 z - a^2$$

$$x^2 - a^2 = a^2 (\sec^2 z - 1)$$

$$x^2 - a^2 = a^2 (\operatorname{tg}^2 z)$$

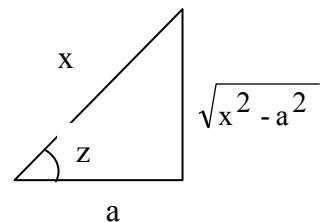
$$\text{si } x = a \sec z \Rightarrow \sec z = \frac{x}{a}$$

$$z = \operatorname{arc sec} \left(\frac{x}{a} \right)$$

$$\text{si } \sec z = \frac{x}{a} \Rightarrow \cos z = \frac{a}{x}$$

$$\text{si } \operatorname{tg} z = \frac{\sqrt{x^2 - a^2}}{a} \Rightarrow \cot z = \frac{a}{\sqrt{x^2 - a^2}}$$

$$\text{si } \sin z = \frac{\sqrt{x^2 - a^2}}{x} \Rightarrow \csc z = \frac{x}{\sqrt{x^2 - a^2}}$$



Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsen\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{a \cos z \ dz}{a \cos z}$$

$$\int \frac{a \cos z \ dz}{a \cos z} = \int dz$$

Tabla de integrales

$$\int dz = z + c$$

Reemplazando

$$z + c = \arcsen\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsen\left(\frac{x}{a}\right) + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sen z$$

$$x^2 = a^2 \sen^2 z$$

$$\text{Si } x = a \sen z \rightarrow dx = a \cos z dz$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sen^2 z}$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2(1 - \sen^2 z)}$$

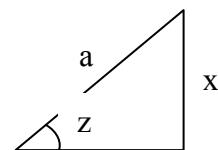
$$\sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2 z}$$

$$\sqrt{a^2 - x^2} = a \cos z$$

$$\text{si } x = a \sen z \Rightarrow \sen z = \frac{x}{a}$$

$$z = \arcsen\left(\frac{x}{a}\right)$$

$$\operatorname{tg} z = \frac{x}{\sqrt{a^2 - x^2}}$$



$$\sqrt{a^2 - x^2}$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{\sqrt{x^2 - a^2}}$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \int \frac{a \sec z \tan z \ dz}{(a \tan z)}$$

$$\int \sec z \ dz =$$

Tabla de integrales

$$\int \sec z \ dz = \ln |\sec z + \tan z| + c$$

Reemplazando

$$\int \sec z \ dz = \ln |\sec z + \tan z| + c$$

$$\int \sec z \ dz = \ln \left| \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right| + c$$

$$\int \sec z \ dz = \ln \left| \frac{x + \sqrt{x^2 - a^2}}{a} \right| + c$$

$$\int \sec z \ dz = \ln \left| x + \sqrt{x^2 - a^2} \right| - \ln |a| + c$$

Cancelando términos semejantes

$$\text{Pero } c_1 = -\ln |a| + c$$

$$\int \sec z \ dz = \ln \left| x + \sqrt{x^2 - a^2} \right| + c_1$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$x = a \sec z$$

$$x^2 = a^2 \sec^2 z$$

$$\text{si } x = a \sec z \Rightarrow dx = a \sec z \tan z \ dz$$

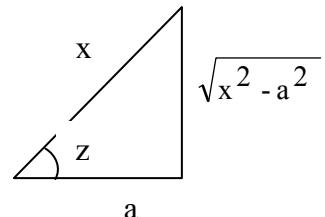
$$\sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2 z - a^2}$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 (\sec^2 z - 1)}$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 (\tan^2 z)}$$

$$\sqrt{x^2 - a^2} = a \tan z$$

$$\sec z = \frac{x}{a} \quad \cos z = \frac{a}{x}$$



$$\int \frac{dx}{x^2 - a^2} = \ln \left| x + \sqrt{x^2 - a^2} \right| + c$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + C \quad \text{donde } a > 0$$

$$\int \frac{dx}{x \sqrt{x^2 - a^2}}$$

$$\int \frac{dx}{x \sqrt{x^2 - a^2}} = \int \frac{a \sec z \, \operatorname{tg} z \, dz}{(a \sec z) a \operatorname{tg} z} = \int \frac{1}{a} dz$$

$$\frac{1}{a} \int dz =$$

Tabla de integrales

$$\int dz = z + C$$

$$\frac{1}{a} \int dz = \frac{1}{a} (z) + C$$

Reemplazando

$$\frac{1}{a} (z) + C = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + C$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$x^2 = a^2 \sec^2 z$$

$$\text{Si } x = a \sec z \rightarrow dx = a \sec z \operatorname{tg} z dz$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2 z - a^2}$$

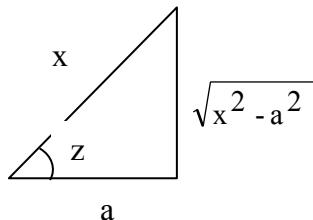
$$\sqrt{x^2 - a^2} = \sqrt{a^2 (\sec^2 z - 1)}$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 (\operatorname{tg}^2 z)}$$

$$\sqrt{x^2 - a^2} = a \operatorname{tg} z$$

$$\text{si } x = a \sec z \Rightarrow \sec z = \frac{x}{a}$$

$$z = \operatorname{arcsec}\left(\frac{x}{a}\right)$$



Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \sqrt{a^2 + x^2} dx$$

$$\int \sqrt{a^2 + x^2} dx = \int a \sec z (a \sec^2 z dz) = \int a^2 \sec^3 z dz$$

$$a^2 \int \sec^3 z dz =$$

Se resuelve la integral $a^2 \int \sec^3 z dz$ por partes

$$\int u dv = u * v - \int v du$$

$$a^2 \int \sec^3 z dz = a^2 \int \sec^2 z * \sec z dz$$

Se resuelve por partes

$$u = \sec z \quad dv = \sec^2 z$$

$$du = \sec z \tan z dz \quad \int dv = \int \sec^2 z dz \\ v = \tan z$$

$$\int u dv = u * v - \int v du$$

$$a^2 \int \sec^2 z * \sec z dz = a^2 [\sec z * \tan z - \int (\tan z)^2 * \sec z dz]$$

$$a^2 [\sec z * \tan z - \int \tan^2 z * \sec z dz]$$

Reemplazando la Identidad trigonométrica

$$\tan^2 z = \sec^2 z - 1$$

$$a^2 [\sec z * \tan z - \int (\sec^2 z - 1) * \sec z dz]$$

$$a^2 [\sec z * \tan z - \int \sec^3 z dz + \int \sec z dz]$$

$$a^2 \int \sec^3 z dz = a^2 [\sec z * \tan z - \int \sec^3 z dz + \int \sec z dz]$$

$$a^2 \int \sec^3 z dz = a^2 \sec z * \tan z - a^2 \int \sec^3 z dz + a^2 \int \sec z dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$a^2 \int \sec^3 z dz + a^2 \int \sec^3 z dz = a^2 \sec z * \tan z + a^2 \int \sec z dz$$

$$2a^2 \int \sec^3 z dz = a^2 \sec z * \tan z + a^2 \int \sec z dz$$

Dividiendo la ecuación por 2

$$\sqrt{a^2 + x^2} \Rightarrow x = a \tan z$$

$$x = a \tan z$$

$$x^2 = a^2 \tan^2 z$$

$$\text{si } x = a \tan z \Rightarrow dx = a \sec^2 z dz$$

$$\sqrt{a^2 + x^2} = \sqrt{a^2 + a^2 \tan^2 z}$$

$$\sqrt{a^2 + x^2} = \sqrt{a^2 (1 + \tan^2 z)}$$

$$\sqrt{a^2 + x^2} = \sqrt{a^2 (\sec^2 z)}$$

$$\sqrt{a^2 + x^2} = a \sec z$$

$$a^2 \int \sec^3 z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + c$$

$$a^2 \int \sec^3 z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \ln |\sec z + \operatorname{tg} z| + c$$

$$\int \sqrt{a^2 + x^2} dx = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \ln |\sec z + \operatorname{tg} z| + c$$

$$\int \sqrt{a^2 + x^2} dx = \frac{a^2}{2} \left(\frac{\sqrt{a^2 + x^2}}{a} \right) * \left(\frac{x}{a} \right) + \frac{a^2}{2} \ln \left| \frac{\sqrt{a^2 + x^2}}{a} + \frac{x}{a} \right| + c$$

$$\int \sqrt{a^2 + x^2} dx = \frac{a^2}{2} \left(\frac{x \sqrt{a^2 + x^2}}{a^2} \right) + \frac{a^2}{2} \ln \left| \frac{\sqrt{a^2 + x^2} + x}{a} \right| + c$$

$$\int \sqrt{a^2 + x^2} dx = \frac{1}{2} \left(x \sqrt{a^2 + x^2} \right) + \frac{a^2}{2} \ln \left| \frac{\sqrt{a^2 + x^2} + x}{a} \right| + c$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{a}$$

$$\cos z = \frac{a}{\sqrt{a^2 + x^2}}$$

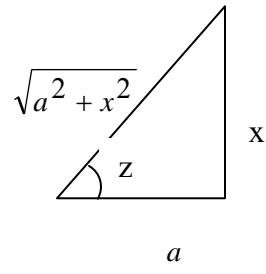
$$\sqrt{a^2 + x^2} = a \sec z$$

$$\sec z = \frac{\sqrt{a^2 + x^2}}{a}$$

Propiedad de los logaritmos

$$\int \sqrt{a^2 + x^2} dx = \frac{1}{2} \left(x \sqrt{a^2 + x^2} \right) + \frac{a^2}{2} \ln \left| \sqrt{a^2 + x^2} + x \right| - \frac{a^2}{2} \ln |a| + c$$

$$C_1 = - \frac{a^2}{2} \ln |a| + c$$



$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln \left| \sqrt{a^2 + x^2} + x \right| + C_1$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \sqrt{x^2 - a^2} dx$$

$$\int \sqrt{x^2 - a^2} dx = \int a \operatorname{tg} z (a \sec z \operatorname{tg} z dz) = \int a^2 \sec z \operatorname{tg}^2 z dz$$

$$a^2 \int \operatorname{tg}^2 z \sec z dz =$$

Identidades trigonométricas

$$\operatorname{tg}^2 z = \sec^2 z - 1$$

$$a^2 \int \operatorname{tg}^2 z \sec z dz = a^2 \int (\sec^2 z - 1) \sec z dz$$

$$a^2 \int (\sec^2 z - 1) \sec z dz = a^2 \int \sec^2 z \sec z dz - a^2 \int \sec z dz$$

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz =$$

Se resuelve la integral $a^2 \int \sec^3 z dz$ por partes

$$\int u dv = u * v - \int v du$$

$$a^2 \int \sec^3 z dz = a^2 \int \sec^2 z * \sec z dz$$

Se resuelve por partes

$u = \sec z$	$dv = \sec^2 z$
$du = \sec z \operatorname{tg} z dz$	$\int dv = \int \sec^2 z dz$
	$v = \operatorname{ta} z$

$$\int u dv = u * v - \int v du$$

$$a^2 \int \sec^2 z * \sec z dz = a^2 [\sec z * \operatorname{tg} z - \int (\operatorname{tg} z) * \sec z * \operatorname{tg} z dz]$$

$$a^2 [\sec z * \operatorname{tg} z - \int \operatorname{tg}^2 z * \sec z dz]$$

Reemplazando la Identidad trigonométrica
 $\operatorname{tg}^2 z = \sec^2 z - 1$

$$a^2 [\sec z * \operatorname{tg} z - \int (\sec^2 z - 1) * \sec z dz]$$

$$a^2 [\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz]$$

$$a^2 \int \sec^3 z dz = a^2 [\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz]$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$x = a \sec z$$

$$x^2 = a^2 \sec^2 z$$

$$\text{si } x = a \sec z \Rightarrow dx = a \sec z \operatorname{tg} z dz$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2 z - a^2}$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 (\sec^2 z - 1)}$$

$$\sqrt{x^2 - a^2} = \sqrt{a^2 (\operatorname{tg}^2 z)}$$

$$\sqrt{x^2 - a^2} = a \operatorname{tg} z$$

$$a^2 \int \sec^3 z dz = a^2 \sec z * \operatorname{tg} z - a^2 \int \sec^3 z dz + a^2 \int \sec z dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$a^2 \int \sec^3 z dz + a^2 \int \sec^3 z dz = a^2 \sec z * \operatorname{tg} z + a^2 \int \sec z dz$$

$$2a^2 \int \sec^3 z dz = a^2 \sec z * \operatorname{tg} z + a^2 \int \sec z dz$$

Dividiendo la ecuación por 2

$$a^2 \int \sec^3 z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + c$$

Regresando a la integral inicial después de resolver $a^2 \int \sec^3 z dz$ por partes.

$$a^2 \int \sec^3 z dz - \left(a^2 \int \sec z dz \right) = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \int \sec z dz - \left(a^2 \int \sec z dz \right)$$

$$\frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \int \sec z dz - a^2 \int \sec z dz$$

Se reducen términos semejantes

$$\frac{a^2}{2} \sec z * \operatorname{tg} z - \frac{a^2}{2} \int \sec z dz$$

$$x = a \sec z$$

$$\text{si } x = a \sec z \Rightarrow \sec z = \frac{x}{a}$$

$$\cos z = \frac{a}{x}$$

$$\sqrt{x^2 - a^2} = a \operatorname{tg} z$$

$$\operatorname{tg} z = \frac{\sqrt{x^2 - a^2}}{a}$$

aplicando la tabla de integrales

$$\frac{a^2}{2} \sec z * \operatorname{tg} z - \frac{a^2}{2} \ln |\sec z + \operatorname{tg} z| + c$$

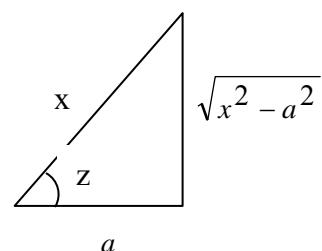
Reemplazando

$$\int \sqrt{x^2 - a^2} dz = \frac{a^2}{2} \sec z * \operatorname{tg} z - \frac{a^2}{2} \ln |\sec z + \operatorname{tg} z| + c$$

$$\int \sqrt{x^2 - a^2} dz = \frac{a^2}{2} \left(\frac{x}{a} \right) * \left(\frac{\sqrt{x^2 - a^2}}{a} \right) - \frac{a^2}{2} \ln \left| \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right| + c$$

$$\int \sqrt{x^2 - a^2} dz = \frac{a^2}{2} \left(\frac{x \sqrt{x^2 - a^2}}{a^2} \right) - \frac{a^2}{2} \ln \left| \frac{x + \sqrt{x^2 - a^2}}{a} \right| + c$$

$$\int \sqrt{x^2 - a^2} dz = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| \frac{x + \sqrt{x^2 - a^2}}{a} \right| + c$$



Propiedad de los logaritmos

$$\int \sqrt{x^2 - a^2} dz = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + \frac{a^2}{2} \ln |a| + c$$

Pero:

$$C_1 = \frac{a^2}{2} \ln |a| + c$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \ln \left| \sqrt{x^2 - a^2} + x \right| + C_1$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \sqrt{a^2 - x^2} dx$$

$$\int \sqrt{a^2 - x^2} dx = \int a \cos z * (a \cos z dz) = \int a^2 \cos^2 z dz$$

Reemplazando la Identidad trigonométrica

$$\cos^2 z = \frac{1}{2}(1 + \cos 2z)$$

$$a^2 \int \cos^2 z dz = a^2 \int \frac{1}{2}(1 + \cos 2z) dz = \frac{a^2}{2} \int (1 + \cos 2z) dz$$

$$\frac{a^2}{2} \int dz + \frac{a^2}{2} \int \cos 2z dz$$

$$\frac{a^2}{2}z + \frac{a^2}{2} \left(\frac{1}{2} \sin 2z \right) + C$$

$$\frac{a^2}{2}z + \left(\frac{a^2}{4} \sin 2z \right) + C$$

Reemplazando la Identidad trigonométrica

$$\sin 2z = 2 \sin z \cos z$$

$$\frac{a^2}{2}z + \left(\frac{a^2}{4} \sin 2z \right) + C = \frac{a^2}{2}z + \frac{a^2}{4}(2 \sin z \cos z) + C$$

Reemplazando

$$\frac{a^2}{2} \left[\arcsin \left(\frac{x}{a} \right) \right] + \frac{a^2}{2} \left(\frac{x}{a} * \frac{\sqrt{a^2 - x^2}}{a} \right)$$

$$\frac{a^2}{2} \arcsin \frac{x}{a} + \frac{a^2}{2} \left(\frac{x * \sqrt{a^2 - x^2}}{a^2} \right) + C = \frac{a^2}{2} \arcsin \frac{x}{a} + \frac{1}{2} \left(x \sqrt{a^2 - x^2} \right) + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{a^2}{2} \arcsin \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + C$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$x^2 = a^2 \sin^2 z$$

$$\text{Si } x = a \sin z \rightarrow dx = a \cos z dz$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2 z}$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2(1 - \sin^2 z)}$$

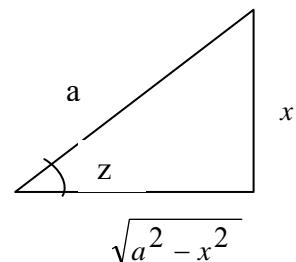
$$\sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2 z}$$

$$\sqrt{a^2 - x^2} = a \cos z$$

$$\text{si } x = a \sin z \Rightarrow \sin z = \frac{x}{a}$$

$$z = \arcsin \left(\frac{x}{a} \right) \quad \cos z = \frac{\sqrt{a^2 - x^2}}{a}$$

$$\tan z = \frac{x}{\sqrt{a^2 - x^2}}$$



Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{x \sqrt{x^2 + a^2}}$$

$$\int \frac{dx}{x \sqrt{x^2 + a^2}} = \int \frac{a \sec^2 z dz}{a \operatorname{tg} z (a \sec z)}$$

$$\frac{1}{a} \int \frac{\sec z dz}{\operatorname{tg} z}$$

$$\frac{1}{a} \int \frac{\frac{1}{\cos z} dz}{\frac{\sin z}{\cos z}} = \frac{1}{a} \int \frac{1}{\sin z} dz$$

$$\frac{1}{a} \int \csc z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \operatorname{ctg} z| + c$$

$$\frac{1}{a} \int \csc z dz = \frac{1}{a} \ln |\csc z - \operatorname{ctg} z| + c$$

Reemplazando

$$\frac{1}{a} \ln |\csc z - \operatorname{ctg} z| + c = \frac{1}{a} \ln \left| \frac{\sqrt{x^2 + a^2}}{x} - \frac{a}{x} \right| + c$$

$$\int \frac{dx}{x \sqrt{x^2 + a^2}} = \frac{1}{a} \ln \left| \frac{\sqrt{x^2 + a^2}}{x} - a \right| + c$$

$$\sqrt{x^2 + a^2} \Rightarrow x = a \operatorname{tg} z$$

$$x = a \operatorname{tg} z$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow dx = a \sec^2 z dz$$

$$\sqrt{x^2 + a^2} = \sqrt{a^2 \operatorname{tg}^2 z + a^2}$$

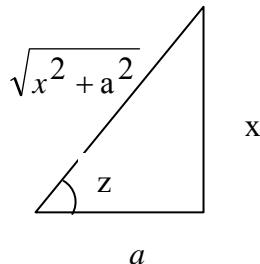
$$\sqrt{x^2 + a^2} = \sqrt{a^2 (\operatorname{tg}^2 z + 1)}$$

$$\sqrt{x^2 + a^2} = \sqrt{a^2 (\sec^2 z)}$$

$$\sqrt{x^2 + a^2} = a \sec z$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{a}$$

$$\csc z = \frac{\sqrt{x^2 + a^2}}{x} \quad \operatorname{ctg} z = \frac{a}{x}$$



Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = \int \frac{a^2 \operatorname{tg}^2 z (a \sec^2 z dz)}{a \sec z}$$

$$a^2 \int \operatorname{tg}^2 z \sec z dz$$

Identidad trigonométrica

$$\operatorname{tg}^2 z = \sec^2 z - 1$$

$$a^2 \int \operatorname{tg}^2 z \sec z dz = a^2 \int (\sec^2 z - 1) (\sec z dz)$$

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = a^2 \int \sec^3 z dz - a^2 \int \sec z dz$$

Esta solución se resuelve primero la integral $a^2 \int \sec^3 z dz$ por partes y después la otra integral

$$\int u dv = u * v - \int v du$$

$$a^2 \int \sec^3 z dz = a^2 \int \sec^2 z * \sec z dz$$

Se resuelve por partes

$$u = \sec z$$

$$dv = \sec^2 z$$

$$du = \sec z \operatorname{tg} z dz$$

$$\int dv = \int \sec^2 z dz$$

$$v = \operatorname{tg} z$$

$$\int u dv = u * v - \int v du$$

$$a^2 \int \sec^3 z dz = a^2 \int \sec^2 z * \sec z dz = a^2 [\sec z * \operatorname{tg} z - \int (\operatorname{tg} z)^2 * \sec z dz]$$

$$a^2 \int \sec^3 z dz = a^2 [\sec z * \operatorname{tg} z - \int \operatorname{tg}^2 z * \sec z dz]$$

Reemplazando la Identidad trigonométrica $\operatorname{tg}^2 z = \sec^2 z - 1$

$$a^2 [\sec z * \operatorname{tg} z - \int (\sec^2 z - 1) * \sec z dz] = a^2 [\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz]$$

$$\sqrt{x^2 + a^2} \Rightarrow x = a \operatorname{tg} z$$

$$x = a \operatorname{tg} z$$

$$x^2 = a^2 \operatorname{tg}^2 z$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow dx = a \sec^2 z dz$$

$$\sqrt{x^2 + a^2} = \sqrt{a^2 \operatorname{tg}^2 z + a^2}$$

$$\sqrt{x^2 + a^2} = \sqrt{a^2 (\operatorname{tg}^2 z + 1)}$$

$$\sqrt{x^2 + a^2} = \sqrt{a^2 (\sec^2 z)}$$

$$\sqrt{x^2 + a^2} = a \sec z$$

$$\text{si } x = a \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{a}$$

$$a^2 \int \sec^3 z dz = a^2 \left[\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz \right]$$

$$a^2 \int \sec^3 z dz = a^2 \sec z * \operatorname{tg} z - a^2 \int \sec^3 z dz + a^2 \int \sec z dz$$

ordenando como una ecuación cualquiera y reduciendo términos semejantes

$$a^2 \int \sec^3 z dz + a^2 \int \sec^3 z dz = a^2 \sec z * \operatorname{tg} z + a^2 \int \sec z dz$$

$$2a^2 \int \sec^3 z dz = a^2 \sec z * \operatorname{tg} z + a^2 \int \sec z dz$$

Dividiendo la ecuación por 2 se obtiene la primera parte de la solución

$$a^2 \int \sec^3 z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \int \sec z dz$$

si	$x = a \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{a}$
$\cos z = \frac{a}{\sqrt{x^2 + a^2}}$	$\sec z = \frac{\sqrt{x^2 + a^2}}{a}$
$\operatorname{ctg} z = \frac{a}{x}$	

La integral inicial es :

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = a^2 \int \sec^3 z dz - a^2 \int \sec z dz$$

Se reemplaza en la integral inicial y se sigue resolviendo la integral

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z + \frac{a^2}{2} \int \sec z dz - a^2 \int \sec z dz$$

Reduciendo términos semejantes

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z - \frac{1}{2} a^2 \int \sec z dz + c$$

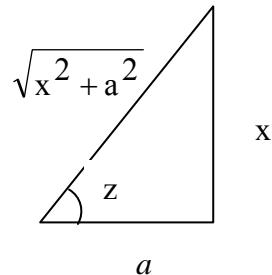


Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + c$$

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z - \frac{a^2}{2} \ln |\sec z + \operatorname{tg} z| + c$$

Reemplazando

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{a^2}{2} \sec z * \operatorname{tg} z - \frac{a^2}{2} \ln |\sec z + \operatorname{tg} z| + c$$

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{a^2}{2} \left(\frac{\sqrt{x^2 + a^2}}{a} \right) * \frac{x}{a} - \frac{a^2}{2} \ln \left| \frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right| + c$$

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{a^2}{2} \left(\frac{x \sqrt{x^2 + a^2}}{a^2} \right) - \frac{a^2}{2} \ln \left| \frac{\sqrt{x^2 + a^2} + x}{a} \right| + c$$

$$a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln \left| \frac{\sqrt{x^2 + a^2} + x}{a} \right| + c$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln \left| \sqrt{x^2 + a^2} + x \right| + \frac{a^2}{2} \ln |a| + c$$

Pero: $C_1 = \frac{1}{2} a^2 \ln |a| + c$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = a^2 \int \sec^3 z dz - a^2 \int \sec z dz = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln \left| \sqrt{x^2 + a^2} + x \right| + c_1$$

Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}}$$

$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = \int \frac{a \cos z \ dz}{a^2 \sin^2 z (a \cos z)}$$

$$\frac{1}{a^2} \int \frac{1}{\sin^2 z} dz$$

$$\frac{1}{a^2} \int \csc^2 z dz$$

Tabla de integrales

$$\int \csc^2 z dz = -\operatorname{ctg} z + c$$

$$\frac{1}{a^2} \int \csc^2 z dz = -\frac{1}{a^2} \operatorname{ctg} z + c$$

Reemplazando

$$-\frac{1}{a^2} (\operatorname{ctg} z) + c = -\frac{1}{a^2} \left(\frac{\sqrt{a^2 - x^2}}{x} \right) + c$$

$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$x = a \sin z$$

$$x^2 = a^2 \sin^2 z$$

$$\text{si } x = a \sin z \Rightarrow dx = a \cos z dz$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2 z}$$

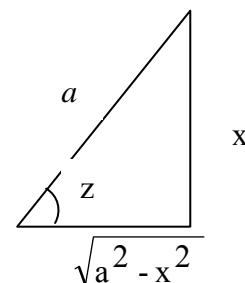
$$\sqrt{a^2 - x^2} = \sqrt{a^2 (1 - \sin^2 z)}$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2 z}$$

$$\sqrt{a^2 - x^2} = a \cos z$$

$$\text{si } x = a \sin z \Rightarrow \sin z = \frac{x}{a}$$

$$\operatorname{tg} z = \frac{x}{\sqrt{a^2 - x^2}} \quad \operatorname{ctg} z = \frac{\sqrt{a^2 - x^2}}{x}$$



Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = \int \frac{(a \cos z)(a \cos z dz)}{a^2 \sin^2 z}$$

$$\int \frac{a^2 \cos^2 z}{a^2 \sin^2 z} dz$$

$$\int \operatorname{ctg}^2 z dz$$

Identidad trigonométrica

$$\operatorname{ctg}^2 z = \csc^2 z - 1$$

$$\int \operatorname{ctg}^2 z dz = \int (\csc^2 z - 1) dz$$

$$\int (\csc^2 z - 1) dz = \int \csc^2 z dz - \int dz$$

Tabla de integrales

$$\int \csc^2 z dz = -\operatorname{ctg} z + c$$

Reemplazando

$$\int \csc^2 z dz - \int dz = -\operatorname{ctg} z - z + c$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \operatorname{arc sen} \left(\frac{x}{a} \right) + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$x = a \sin z$$

$$x^2 = a^2 \sin^2 z$$

$$\text{si } x = a \sin z \Rightarrow dx = a \cos z dz$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2 z}$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 (1 - \sin^2 z)}$$

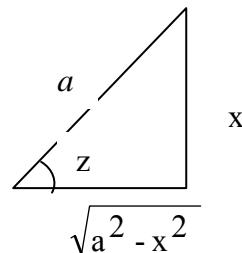
$$\sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2 z}$$

$$\sqrt{a^2 - x^2} = a \cos z$$

$$\text{si } x = a \sin z \Rightarrow \sin z = \frac{x}{a}$$

$$z = \operatorname{arc sen} \left(\frac{x}{a} \right)$$

$$\operatorname{tg} z = \frac{x}{\sqrt{a^2 - x^2}} \quad \operatorname{ctg} z = \frac{\sqrt{a^2 - x^2}}{x}$$



Encuentre una formula directa para la aplicación de la siguiente integral

$$\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}}$$

$$\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}} = \int \frac{(a^2 \sin^2 z)(a \cos z dz)}{a^3 \cos^3 z} = \int \frac{\sin^2 z dz}{\cos^2 z}$$

$$\int \tan^2 z dz$$

Identidad trigonométrica

$$\tan^2 z = \sec^2 z - 1$$

$$\int \tan^2 z dz = \int (\sec^2 z - 1) dz = \int \sec^2 z dz - \int dz$$

Tabla de integrales

$$\int \sec^2 z dz = \tan z + c$$

$$\int dz = z + c$$

$$\int \sec^2 z dz - \int dz = \tan z - z + c$$

Reemplazando

$$\tan z - z + c = \frac{x}{\sqrt{a^2 - x^2}} - \arcsin \frac{x}{a} + c$$

$$\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}} = \frac{x}{\sqrt{a^2 - x^2}} - \arcsin \frac{x}{a} + c$$

$$(a^2 - x^2)^{3/2} \Rightarrow x = a \sin z$$

$$x^2 = a^2 \sin^2 z$$

$$\text{si } x = a \sin z \Rightarrow dx = a \cos z dz$$

$$(a^2 - x^2)^{3/2} = (a^2 - a^2 \sin^2 z)^{3/2}$$

$$(a^2 - x^2)^{3/2} = [a^2(1 - \sin^2 z)]^{3/2}$$

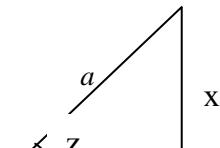
$$(a^2 - x^2)^{3/2} = [a^2(\cos^2 z)]^{3/2}$$

$$(a^2 - x^2)^{3/2} = a^3 \cos^3 z$$

$$\text{si } x = a \sin z \Rightarrow \sin z = \frac{x}{a}$$

$$z = \arcsin \frac{x}{a}$$

$$\tan z = \frac{x}{\sqrt{a^2 - x^2}}$$



$$\sqrt{a^2 - x^2}$$

INTEGRALES POR SUSTITUCIONES TRIGONOMETRICAS

Ejercicios 7.3 Pag 571 Leythold

Problema # 1

$$\int \frac{dx}{x^2 \sqrt{4-x^2}}$$

$$\int \frac{dx}{x^2 \sqrt{4-x^2}} = \int \frac{2 \cos z \ dz}{4 \sin^2 z (2 \cos z)}$$

$$\frac{1}{4} \int \frac{dz}{\sin^2 z}$$

$$\frac{1}{4} \int \frac{1}{\sin^2 z} dz = \frac{1}{4} \int \csc^2 z dz$$

Tabla de integrales

$$\int \csc^2 z dz = -\operatorname{ctg} z + c$$

$$\frac{1}{4} \int \csc^2 z dz = -\frac{1}{4} \operatorname{ctg} z + c$$

Reemplazando

$$-\frac{1}{4}(\operatorname{ctg} z) + c = -\frac{1}{4}\left(\frac{\sqrt{4-x^2}}{x}\right) + c$$

$$\int \frac{dx}{x^2 \sqrt{4-x^2}} = -\frac{\sqrt{4-x^2}}{4x} + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\begin{aligned} \sqrt{4-x^2} &= \sqrt{2^2 - x^2} \\ \sqrt{2^2 - x^2} &\Rightarrow x = 2 \sin z \end{aligned}$$

$$x = 2 \sin z$$

$$x^2 = 4 \sin^2 z$$

$$\text{si } x = 2 \sin z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4-x^2} = \sqrt{4 - 4 \sin^2 z}$$

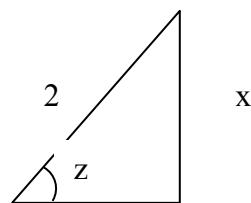
$$\sqrt{4-x^2} = \sqrt{4(1 - \sin^2 z)}$$

$$\sqrt{4-x^2} = \sqrt{4(\cos^2 z)}$$

$$\sqrt{4-x^2} = 2 \cos z$$

$$\text{si } x = 2 \sin z \Rightarrow \sin z = \frac{x}{2}$$

$$\operatorname{tg} z = \frac{x}{\sqrt{4-x^2}} \quad \operatorname{ctg} z = \frac{\sqrt{4-x^2}}{x}$$



$$\sqrt{4-x^2}$$

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Problema # 2

$$\int \frac{\sqrt{4 - x^2}}{x^2} dx$$

$$\int \frac{\sqrt{4 - x^2}}{x^2} dx = \int \frac{2 \cos z (2 \cos z dz)}{4 \sin^2 z}$$

$$\int \frac{\cos^2 z}{\sin^2 z} dz$$

$$\int \operatorname{ctg}^2 z dz$$

Identidad trigonométrica
 $\operatorname{ctg}^2 z = \csc^2 z - 1$

$$\int \operatorname{ctg}^2 z dz = \int (\csc^2 z - 1) dz$$

$$\int (\csc^2 z - 1) dz = \int \csc^2 z dz - \int 1 dz$$

Tabla de integrales

$$\int dz = z + c$$

$$\int \csc^2 z dz = -\operatorname{ctg} z + c$$

$$\int \csc^2 z dz - \int 1 dz = -\operatorname{ctg} z - z + c$$

Reemplazando

$$-\operatorname{ctg} z - z + c = -\left(\frac{\sqrt{4 - x^2}}{x}\right) - \operatorname{arc sen}\left(\frac{x}{2}\right) + c$$

$$\int \frac{\sqrt{4 - x^2}}{x^2} dx = -\left[\frac{\sqrt{4 - x^2}}{x}\right] - \operatorname{arc sen}\left[\frac{x}{2}\right] + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \operatorname{sen} z$$

$$\sqrt{4 - x^2} = \sqrt{2^2 - x^2}$$

$$\sqrt{2^2 - x^2} \Rightarrow x = 2 \operatorname{sen} z$$

$$x = 2 \operatorname{sen} z$$

$$x^2 = 4 \operatorname{sen}^2 z$$

$$\text{si } x = 2 \operatorname{sen} z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4 - x^2} = \sqrt{4 - 4 \operatorname{sen}^2 z}$$

$$\sqrt{4 - x^2} = \sqrt{4(1 - \operatorname{sen}^2 z)}$$

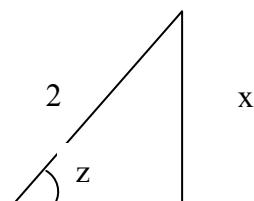
$$\sqrt{4 - x^2} = \sqrt{4(\cos^2 z)}$$

$$\sqrt{4 - x^2} = 2 \cos z$$

$$\text{si } x = 2 \operatorname{sen} z \Rightarrow \operatorname{sen} z = \frac{x}{2}$$

$$\operatorname{sen} z = \frac{x}{2} \Rightarrow z = \operatorname{arc sen}\left(\frac{x}{2}\right)$$

$$\operatorname{tg} z = \frac{x}{\sqrt{4 - x^2}} \quad \operatorname{ctg} z = \frac{\sqrt{4 - x^2}}{x}$$



$$\sqrt{4 - x^2}$$

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Problema # 3

$$\int \frac{dx}{x \sqrt{x^2 + 4}}$$

$$\int \frac{dx}{x \sqrt{x^2 + 4}} = \int \frac{2 \sec^2 z dz}{2 \operatorname{tg} z (2 \sec z)}$$

$$\frac{1}{2} \int \frac{\sec z dz}{\operatorname{tg} z}$$

$$\frac{1}{2} \int \frac{\cos z}{\frac{\sin z}{\cos z}} dz$$

$$\frac{1}{2} \int \frac{1}{\sin z} dz$$

$$\frac{1}{2} \int \csc z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \operatorname{ctg} z| + c$$

$$\frac{1}{2} \int \csc z dz = \frac{1}{2} \ln |\csc z - \operatorname{ctg} z| + c$$

Reemplazando

$$\frac{1}{2} \ln |\csc z - \operatorname{ctg} z| + c = \frac{1}{2} \ln \left| \frac{\sqrt{x^2 + 4}}{x} - \frac{2}{x} \right| + c$$

$$\int \frac{dx}{x \sqrt{x^2 + 4}} = \frac{1}{2} \ln \left| \frac{\sqrt{x^2 + 4}}{x} - 2 \right| + c$$

$$\sqrt{x^2 + a^2} \Rightarrow x = a \operatorname{tg} z$$

$$\sqrt{x^2 + 4} = \sqrt{x^2 + 2^2} \Rightarrow x = 2 \operatorname{tg} z$$

$$x = 2 \operatorname{tg} z \\ x^2 = 4 \operatorname{tg}^2 z$$

$$\text{si } x = 2 \operatorname{tg} z \Rightarrow dx = 2 \sec^2 z dz$$

$$\sqrt{x^2 + 4} = \sqrt{4 \operatorname{tg}^2 z + 4}$$

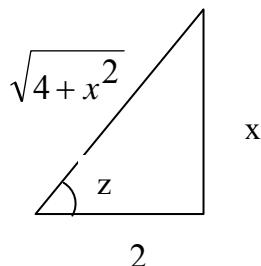
$$\sqrt{x^2 + 4} = \sqrt{4(\operatorname{tg}^2 z + 1)}$$

$$\sqrt{x^2 + 4} = \sqrt{4(\sec^2 z)}$$

$$\sqrt{x^2 + 4} = 2 \sec z$$

$$\text{si } x = 2 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{2}$$

$$\csc z = \frac{\sqrt{x^2 + 4}}{x} \quad \operatorname{ctg} z = \frac{2}{x}$$



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Problema # 4

$$\int \frac{x^2 dx}{\sqrt{x^2 + 6}}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + 6}} = \int \frac{6 \operatorname{tg}^2 z (\sqrt{6} \sec^2 z dz)}{\sqrt{6} \sec z}$$

$$6 \int \operatorname{tg}^2 z \sec z dz$$

Identidad trigonométrica

$$\operatorname{tg}^2 z = \sec^2 z - 1$$

$$6 \int \operatorname{tg}^2 z \sec z dz = 6 \int (\sec^2 z - 1) (\sec z dz)$$

$$6 \int \sec^3 z dz - 6 \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + C$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + 6}} = 6 \int \sec^3 z dz - 6 \int \sec z dz$$

Se resuelve primero la integral $6 \int \sec^3 z dz$ por partes

$$\int u dv = u * v - \int v du$$

$$6 \int \sec^3 z dz = 6 \int \sec^2 z * \sec z dz$$

Se resuelve por partes

$$u = \sec z$$

$$dv = \sec^2 z$$

$$du = \sec z \operatorname{tg} z dz$$

$$\int dv = \int \sec^2 z dz$$

$$v = \operatorname{tg} z$$

$$6 \int \sec^2 z * \sec z dz = 6 [\sec z * \operatorname{tg} z - \int (\operatorname{tg} z)^2 * \sec z dz]$$

$$6 [\sec z * \operatorname{tg} z - \int (\operatorname{tg} z)^2 * \sec z dz] = 6 [\sec z * \operatorname{tg} z - \int \sec^2 z * \sec z dz]$$

Reemplazando la Identidad trigonométrica $\operatorname{tg}^2 z = \sec^2 z - 1$

$$6 [\sec z * \operatorname{tg} z - \int \sec^2 z * \sec z dz] = 6 [\sec z * \operatorname{tg} z - \int (\sec^2 z - 1) * \sec z dz]$$

$$\sqrt{x^2 + 6} \Rightarrow x = a \operatorname{tg} z$$

$$\sqrt{x^2 + 6} = \sqrt{x^2 + (\sqrt{6})^2} \Rightarrow x = \sqrt{6} \operatorname{tg} z$$

$$x = \sqrt{6} \operatorname{tg} z$$

$$x^2 = 6 \operatorname{tg}^2 z$$

$$\text{si } x = \sqrt{6} \operatorname{tg} z \Rightarrow dx = \sqrt{6} \sec^2 z dz$$

$$\sqrt{x^2 + 6} = \sqrt{6 \operatorname{tg}^2 z + 6}$$

$$\sqrt{x^2 + 6} = \sqrt{6(\operatorname{tg}^2 z + 1)}$$

$$\sqrt{x^2 + 6} = \sqrt{6(\sec^2 z)}$$

$$\sqrt{x^2 + 6} = \sqrt{6} \sec z$$

$$\text{si } x = \sqrt{6} \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{\sqrt{6}}$$

$$6 \left[\sec z * \operatorname{tg} z - \int (\sec^2 - 1) * \sec z dz \right] = 6 \left[\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz \right]$$

$$6 \int \sec^3 z dz = 6 \left[\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz \right]$$

$$6 \int \sec^3 z dz = 6 \sec z * \operatorname{tg} z - 6 \int \sec^3 z dz + 6 \int \sec z dz$$

ordenando como una ecuación cualquiera y reduciendo términos semejantes

$$6 \int \sec^3 z dz + 6 \int \sec^3 z dz = 6 \sec z * \operatorname{tg} z + 6 \int \sec z dz$$

$$12 \int \sec^3 z dz = 6 \sec z * \operatorname{tg} z + 6 \int \sec z dz$$

Dividiendo la ecuación por 2

$$6 \int \sec^3 z dz = 3 \sec z * \operatorname{tg} z + 3 \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + c$$

$$6 \int \sec^3 z dz = 3 \sec z * \operatorname{tg} z + 3 \ln |\sec z + \operatorname{tg} z| + c$$

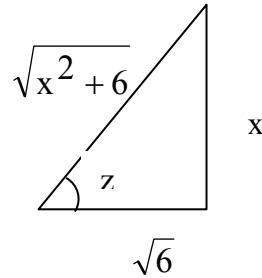
La integral inicial es :

$$\int \frac{x^2 dx}{\sqrt{x^2 + 6}} = 6 \int \sec^3 z dz - 6 \int \sec z dz$$

$$6 \int \sec^3 z dz - 6 \int \sec z dz = 3 \sec z * \operatorname{tg} z + 3 \ln |\sec z + \operatorname{tg} z| - 6 \int \sec z dz$$

$$6 \int \sec^3 z dz - 6 \int \sec z dz = 3 \sec z * \operatorname{tg} z + 3 \ln |\sec z + \operatorname{tg} z| - 6 \ln |\sec z + \operatorname{tg} z| + c$$

$$6 \int \sec^3 z dz - 6 \int \sec z dz = 3 \sec z * \operatorname{tg} z - 3 \ln |\sec z + \operatorname{tg} z| + c$$



$\text{si } x = \sqrt{6} \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{\sqrt{6}}$
$\cos z = \frac{\sqrt{6}}{\sqrt{x^2 + 6}}$
$\sec z = \frac{\sqrt{x^2 + 6}}{\sqrt{6}}$
$\operatorname{ctg} z = \frac{\sqrt{6}}{x}$

Reemplazando

$$6 \int \sec^3 z dz - 6 \int \sec z dz = 3 \sec z * \operatorname{tg} z - 3 \ln |\sec z + \operatorname{tg} z| + c$$

$$6\int \sec^3 z dz - 6\int \sec z dz = 3 \left(\frac{\sqrt{x^2 + 6}}{\sqrt{6}} \right) * \frac{x}{\sqrt{6}} - 3 \ln \left| \frac{\sqrt{x^2 + 6}}{\sqrt{6}} + \frac{x}{\sqrt{6}} \right| + c$$

$$6\int \sec^3 z dz - 6\int \sec z dz = 3 \left(\frac{x \sqrt{x^2 + 6}}{6} \right) - 3 \ln \left| \frac{\sqrt{x^2 + 6} + x}{\sqrt{6}} \right| + c$$

$$6\int \sec^3 z dz - 6\int \sec z dz = \left(\frac{x \sqrt{x^2 + 6}}{2} \right) - 3 \ln \left| \frac{\sqrt{x^2 + 6} + x}{\sqrt{6}} \right| + c$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + 6}} = 6\int \sec^3 z dz - 6\int \sec z dz = \left(\frac{x \sqrt{x^2 + 6}}{2} \right) - 3 \ln \left| \frac{\sqrt{x^2 + 6} + x}{\sqrt{6}} \right| + c$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + 6}} = \left(\frac{x \sqrt{x^2 + 6}}{2} \right) - 3 \ln \left| \frac{\sqrt{x^2 + 6} + x}{\sqrt{6}} \right| + c$$

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Problema # 5

$$\int \frac{x}{\sqrt{x^2 - 25}} dx$$

$$\int \frac{x}{\sqrt{x^2 - 25}} dx = \int \frac{5 \sec z (5 \sec z \tan z dz)}{5 \tan z}$$

$$5 \int \sec^2 z dz =$$

Tabla de integrales

$$\int \sec^2 z dz = \tan z + C$$

$$5 \int \sec^2 z dz = 5 \tan z + C$$

Reemplazando

$$5 (\tan z) + C = 5 \left(\frac{\sqrt{x^2 - 25}}{5} \right) + C$$

$$5 (\tan z) + C = (\sqrt{x^2 - 25}) + C$$

$$\int \frac{x}{\sqrt{x^2 - 25}} dx = (\sqrt{x^2 - 25}) + C$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 25} = \sqrt{x^2 - 5^2} \Rightarrow x = 5 \sec z$$

$$x = 5 \sec z$$

$$x^2 = 25 \sec^2 z$$

$$\text{si } x = 5 \sec z \Rightarrow dx = 5 \sec z \tan z dz$$

$$\sqrt{x^2 - 25} = \sqrt{25 \sec^2 z - 25}$$

$$\sqrt{x^2 - 25} = \sqrt{25(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 25} = \sqrt{25(\tan^2 z)}$$

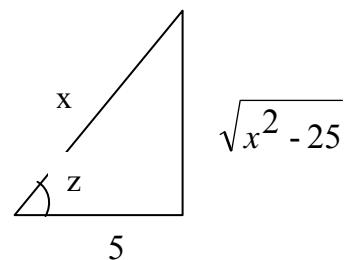
$$\sqrt{x^2 - 25} = 5 \tan z$$

$$\text{si } x = 5 \sec z \Rightarrow \sec z = \frac{x}{5}$$

$$\text{si } \sec z = \frac{x}{5} \Rightarrow \cos z = \frac{5}{x}$$

$$\sin z = \frac{\sqrt{x^2 - 25}}{x}$$

$$\tan z = \frac{\sqrt{x^2 - 25}}{5}$$



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Problema # 6

$$\int \frac{dx}{(2+x^2)^{\frac{3}{2}}}$$

$$\int \frac{dx}{(2+x^2)^{\frac{3}{2}}} = \int \frac{\sqrt{2} \sec^2 z dz}{2\sqrt{2} \sec^3 z}$$

$$\frac{1}{2} \int \frac{1}{\sec} dz$$

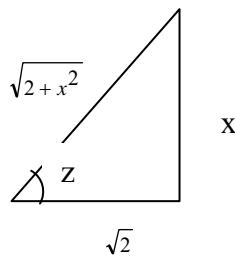
$$\frac{1}{2} \int \cos z dz$$

$$\frac{1}{2} \sin z + c$$

Reemplazando

$$\frac{1}{2} (\sin z) + c = \frac{1}{2} \left(\frac{x}{\sqrt{2+x^2}} \right) + c$$

$$\int \frac{dx}{(2+x^2)^{\frac{3}{2}}} = \frac{x}{2\sqrt{2+x^2}} + c$$



$$(a^2 + x^2)^{3/2} \Rightarrow x = a \tan z$$

$$(2+x^2)^{\frac{3}{2}} = [\sqrt{2}^2 + x^2]^{\frac{3}{2}} \Rightarrow x = \sqrt{2} \tan z$$

$$x = \sqrt{2} \tan z$$

$$x^2 = 2 \tan^2 z$$

$$\text{si } x = \sqrt{2} \tan z \Rightarrow dx = \sqrt{2} \sec^2 z dz$$

$$(2+x^2)^{\frac{3}{2}} = [\sqrt{2}^2 + 2 \tan^2 z]^{\frac{3}{2}}$$

$$(2+x^2)^{\frac{3}{2}} = [2 + 2 \tan^2 z]^{\frac{3}{2}}$$

$$(2+x^2)^{\frac{3}{2}} = [2(1 + \tan^2 z)]^{\frac{3}{2}}$$

$$(2+x^2)^{\frac{3}{2}} = [2(\sec^2 z)]^{\frac{3}{2}}$$

$$(2+x^2)^{\frac{3}{2}} = [2]^{3/2} * [\sec^2 z]^{3/2}$$

$$(2+x^2)^{\frac{3}{2}} = 2\sqrt{2} * \sec^3 z$$

$$\text{si } x = \sqrt{2} \tan z \Rightarrow \tan z = \frac{x}{\sqrt{2}}$$

$$\sin z = \frac{x}{\sqrt{2+x^2}}$$

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Problema # 7

$$\int \frac{dx}{(4x^2 - 9)^{3/2}}$$

$$\int \frac{dx}{(4x^2 - 9)^{3/2}} = \int \frac{\frac{3}{2} \sec z \ tan z dz}{27 \ tan^3 z}$$

$$\frac{1}{18} \int \frac{\sec z dz}{\tan^2 z} = \frac{1}{18} \int \frac{\frac{1}{\cos z}}{\frac{\sin^2 z}{\cos^2 z}} dz$$

$$\frac{1}{18} \int \frac{\cos z}{\sin^2 z} dz$$

$$\frac{1}{18} \int \frac{\cos z}{\sin z} * \frac{1}{\sin z} dz$$

$$\frac{1}{18} \int \cot z \ csc z dz$$

Solución por cambio de variable

$$u = \csc z$$

$$du = -\csc z \ cot z dz$$

$$-\frac{1}{18} \int du = -\frac{1}{18} u + c$$

Reemplazando

$$-\frac{1}{18} u + c = -\frac{1}{18} \csc z + c$$

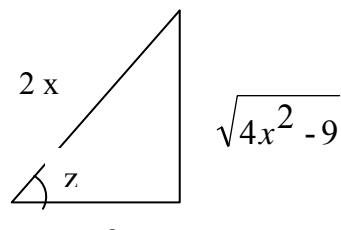
Reemplazando

$$-\frac{1}{18} \csc z + c = -\frac{1}{18} \left(\frac{2x}{\sqrt{4x^2 - 9}} \right) + c$$

$$-\frac{1}{18} \left(\frac{2x}{\sqrt{4x^2 - 9}} \right) + c = -\frac{1}{9} \left[\frac{x}{\sqrt{4x^2 - 9}} \right] + c$$

$$\int \frac{dx}{(4x^2 - 9)^{3/2}} = -\frac{x}{9\sqrt{4x^2 - 9}} + c$$

$$\begin{aligned} (x^2 - a^2)^{3/2} &\Rightarrow x = a \sec z \\ [4x^2 - 9]^{3/2} &= \left(\frac{4x^2}{4} - \frac{9}{4} \right)^{3/2} \\ [4x^2 - 9]^{3/2} &= \left[x^2 - \left(\frac{3}{2} \right)^2 \right]^{3/2} \Rightarrow x = \frac{3}{2} \sec z \\ x &= \frac{3}{2} \sec z \\ x^2 &= \frac{9}{4} \sec^2 z \\ \text{si } x = \frac{3}{2} \sec z \Rightarrow dx &= \frac{3}{2} \sec z \ tan z dz \\ (4x^2 - 9)^{3/2} &= \left[4 \left(\frac{9}{4} \sec^2 z \right) - 9 \right]^{3/2} \\ (4x^2 - 9)^{3/2} &= \left[(9 \sec^2 z) - 9 \right]^{3/2} \\ (4x^2 - 9)^{3/2} &= \left[(9)(\sec^2 z - 1) \right]^{3/2} \\ (4x^2 - 9)^{3/2} &= \left[(9)(\tan^2 z) \right]^{3/2} \\ (4x^2 - 9)^{3/2} &= [3^2]^{3/2} [\tan^2 z]^{3/2} \\ (4x^2 - 9)^{3/2} &= 3^3 \tan^3 z \\ (4x^2 - 9)^{3/2} &= 27 \tan^3 z \\ \sec z &= \frac{2x}{3} \quad \cos z = \frac{3}{2x} \quad \sin z = \frac{\sqrt{4x^2 - 9}}{2x} \\ \csc z &= \frac{2x}{\sqrt{4x^2 - 9}} \end{aligned}$$



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Problema # 8

$$\int \frac{dw}{w^2 \sqrt{w^2 - 7}}$$

$$\int \frac{dw}{w^2 \sqrt{w^2 - 7}} = \int \frac{\sqrt{7} \sec z \ tg z \ dz}{7 \ sec^2 z (\sqrt{7} \ tg z)}$$

$$\frac{1}{7} \int \frac{1}{\sec z} dz$$

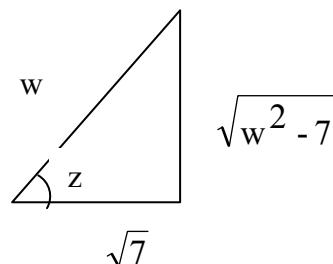
$$\frac{1}{7} \int \cos z dz$$

$$\frac{1}{7} \sin z + c$$

Reemplazando

$$\frac{1}{7} (\sin z) + c = \frac{1}{7} \left(\frac{\sqrt{w^2 - 7}}{w} \right) + c$$

$$\int \frac{dw}{w^2 \sqrt{w^2 - 7}} = \frac{\sqrt{w^2 - 7}}{7w} + c$$



$$\sqrt{w^2 - a^2} \Rightarrow w = a \sec z$$

$$\sqrt{w^2 - 7} = \sqrt{w^2 - \sqrt{7}}$$

$$\sqrt{w^2 - \sqrt{7}} \Rightarrow w = \sqrt{7} \sec z$$

$$w = \sqrt{7} \sec z$$

$$w^2 = 7 \sec^2 z$$

$$\text{si } w = \sqrt{7} \sec z \Rightarrow dw = \sqrt{7} \sec z \ tg z \ dz$$

$$\sqrt{w^2 - 7} = \sqrt{7 \sec^2 z - 7}$$

$$\sqrt{w^2 - 7} = \sqrt{7(\sec^2 z - 1)}$$

$$\sqrt{w^2 - 7} = \sqrt{7(\tg^2 z)}$$

$$\sqrt{w^2 - 7} = \sqrt{7} \tg z$$

$$\text{si } w = \sqrt{7} \sec z \Rightarrow \sec z = \frac{w}{\sqrt{7}}$$

$$\cos z = \frac{\sqrt{7}}{w} \quad \sin z = \frac{\sqrt{w^2 - 7}}{w}$$

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Problema # 9

$$\int \frac{\sec^2 x \, dx}{\left(4 - \tan^2 x\right)^{3/2}}$$

$$\int \frac{\sec^2 x \, dx}{\left(4 - \tan^2 x\right)^{3/2}} = \int \frac{2 \cos z \, dz}{8 \cos^3 z}$$

$$\frac{1}{4} \int \frac{1}{\cos^2 z} \, dz$$

$$\frac{1}{4} \int \sec^2 z \, dz$$

Tabla de integrales

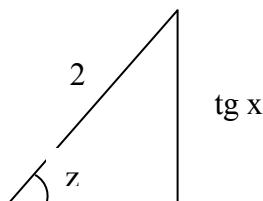
$$\int \sec^2 z \, dz = \tan z + c$$

$$\frac{1}{4} \int \sec^2 z \, dz = \frac{1}{4} \tan z + c$$

Reemplazando

$$\frac{1}{4} (\tan z) + c = \frac{1}{4} \left(\frac{\tan x}{\sqrt{4 - \tan^2 x}} \right) + c$$

$$\int \frac{\sec^2 x \, dx}{\left(4 - \tan^2 x\right)^{3/2}} = \frac{\tan x}{4 \sqrt{4 - \tan^2 x}} + c$$



$$\sqrt{4 - \tan^2 x}$$

$$\left(a^2 - \tan^2 x \right)^{3/2} \Rightarrow \tan x = a \sin z$$

$$\left(4 - \tan^2 x \right)^{3/2} = \left[(2)^2 - \tan^2 x \right]^{3/2}$$

$$\left[(2)^2 - \tan^2 x \right]^{3/2} \Rightarrow \tan x = 2 \sin \theta$$

$$\tan x = 2 \sin z$$

$$\tan^2 x = 4 \sin^2 z$$

$$\text{si } \tan x = 2 \sin z \Rightarrow \sec^2 x \, dx = 2 \cos z \, dz$$

$$\left(4 - \tan^2 x \right)^{3/2} = \left[4 - 4 \sin^2 z \right]^{3/2}$$

$$\left(4 - \tan^2 x \right)^{3/2} = \left[(4)(1 - \sin^2 z) \right]^{3/2}$$

$$\left(4 - \tan^2 x \right)^{3/2} = \left[(4)(\cos^2 z) \right]^{3/2}$$

$$\left(4 - \tan^2 x \right)^{3/2} = \left[(2^2) \right]^{3/2} * \cos^3 z$$

$$\left(4 - \tan^2 x \right)^{3/2} = 2^3 \cos^3 z$$

$$\left(4 - \tan^2 x \right)^{3/2} = 8 \cos^3 z$$

$$\text{si } \tan x = 2 \sin z \Rightarrow \sin z = \frac{\tan x}{2}$$

$$\tan z = \frac{\tan x}{\sqrt{4 - \tan^2 x}}$$

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Problema # 10

$$\int \frac{dz}{(z^2 - 6z + 18)^{3/2}}$$

$$\int \frac{dz}{(z^2 - 6z + 18)^{3/2}} = \int \frac{dz}{(z - 6z + 9 + 9)^{3/2}}$$

$$\int \frac{dz}{[(z - 3)^2 + 9]^{3/2}}$$

Reemplazando

$$\int \frac{dz}{[(z - 3)^2 + 9]^{3/2}} = \int \frac{3 \sec^2 \theta \, d\theta}{27 \sec^3 \theta}$$

$$\frac{1}{9} \int \frac{1}{\sec \theta} \, d\theta$$

$$\frac{1}{9} \int \cos \theta \, d\theta$$

Tabla de integrales

$$\int \cos \theta \, d\theta = \sin \theta + c$$

$$\frac{1}{9} \int \cos \theta \, d\theta = \frac{1}{9} \sin \theta + c$$

Reemplazando

$$\frac{1}{9} \sin \theta + c = \frac{1}{9} \left(\frac{z - 3}{\sqrt{z^2 - 6z + 18}} \right) + c$$

$$\int \frac{dz}{(z^2 - 6z + 18)^{3/2}} = \frac{z - 3}{9(z^2 - 6z + 18)} + c$$

$$(z^2 + a^2)^{3/2} \Rightarrow z = a \tan \theta$$

$$(z - 3)^2 + 9 \stackrel{3}{\overline{\sqrt{}}} = [(z - 3)^2 + 3^2] \stackrel{3}{\overline{\sqrt{}}} \Rightarrow z - 3 = 3 \tan \theta$$

$$z - 3 = 3 \tan \theta$$

$$(z - 3)^2 = 9 \tan^2 \theta$$

$$\text{si } z - 3 = 3 \tan \theta \Rightarrow dz = 3 \sec^2 \theta \, d\theta$$

$$[(z - 3)^2 + 9] \stackrel{3}{\overline{\sqrt{}}} = [9 \tan^2 \theta + 9] \stackrel{3}{\overline{\sqrt{}}}$$

$$[(z - 3)^2 + 9] \stackrel{3}{\overline{\sqrt{}}} = [9(\tan^2 \theta + 1)] \stackrel{3}{\overline{\sqrt{}}}$$

$$[(z - 3)^2 + 9] \stackrel{3}{\overline{\sqrt{}}} = [9(\sec^2 \theta)] \stackrel{3}{\overline{\sqrt{}}}$$

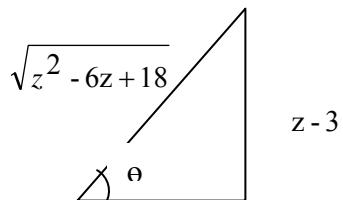
$$[(z - 3)^2 + 9] \stackrel{3}{\overline{\sqrt{}}} = [9]^{3/2} \sec^3 \theta$$

$$[(z - 3)^2 + 9] \stackrel{3}{\overline{\sqrt{}}} = \sqrt{729} \sec^3 \theta$$

$$[(z - 3)^2 + 9] \stackrel{3}{\overline{\sqrt{}}} = 27 \sec^3 \theta$$

$$\text{si } z - 3 = 3 \tan \theta \Rightarrow \tan \theta = \frac{z - 3}{3}$$

$$\tan \theta = \frac{z - 3}{\sqrt{z^2 - 6z + 18}}$$



3

Ejercicios 7.3 Pag 571 Leythold

Problema # 11

$$\int \frac{\ln^3 w \ dw}{w \sqrt{\ln^2 w - 4}}$$

$$\int \frac{\ln^3 w \ dw}{w \sqrt{\ln^2 w - 4}} = \int \frac{\ln^3 w}{\sqrt{\ln^2 w - 4}} \frac{dw}{w}$$

Reemplazando

$$\int \frac{\ln^3 w}{\sqrt{\ln^2 w - 4}} \frac{dw}{w} = \int \frac{8 \sec^3 z (2 \sec z \tan z dz)}{2 \tan \theta}$$

$$8 \int \sec^4 \theta d\theta = 8 \int \sec^2 \theta * \sec^2 \theta d\theta$$

Identidad trigonométrica

$$\tan^2 z + 1 = \sec^2 z$$

Reemplazando

$$8 \int \sec^2 \theta * \sec^2 \theta d\theta = 8 \int (\tan^2 \theta + 1) * \sec^2 \theta d\theta$$

$$8 \int (\tan^2 \theta) \sec^2 \theta d\theta + 8 \int \sec^2 \theta d\theta$$

Solución por cambio de variable

$$x = \tan \theta \quad dx = \sec^2 \theta dz$$

Reemplazando

$$8 \int (\tan^2 \theta) \sec^2 \theta d\theta + 8 \int \sec^2 \theta d\theta = 8 \int x^2 dx + 8 \int dx$$

$$8 \int x^2 dx + 8 \int dx = \frac{8}{3} x^3 + 8x + c$$

$$\frac{8}{3} (\tan \theta)^3 + 8(\tan \theta) + c = \frac{8}{3} \left(\frac{\sqrt{\ln^2 w - 4}}{2} \right)^3 + 8 \left(\frac{\sqrt{\ln^2 w - 4}}{2} \right) + c$$

$$\frac{8}{3} \left(\frac{\sqrt{\ln^2 w - 4}}{(2)^3} \right)^3 + 8 \left(\frac{\sqrt{\ln^2 w - 4}}{2} \right) + c = \frac{8}{3} \frac{\left(\frac{\sqrt{\ln^2 w - 4}}{2} \right)^2 * \left(\sqrt{\ln^2 w - 4} \right)}{8} + 4 \left(\sqrt{\ln^2 w - 4} \right) + c$$

$$\sqrt{u^2 - a^2} \Rightarrow u = a \sec \theta$$

$$\sqrt{\ln^2 w - 4} = \sqrt{\ln^2 w - 2^2} \Rightarrow \ln w = 2 \sec \theta$$

$$\ln w = 2 \sec \theta$$

$$(\ln w)^2 = 4 \sec^2 \theta$$

$$(\ln w)^3 = 8 \sec^3 \theta$$

$$\text{si } \ln w = 2 \sec \theta \Rightarrow \frac{1}{w} dw = 2 \sec \theta \tan \theta d\theta$$

$$\sqrt{\ln^2 w - 4} = \sqrt{4 \sec^2 \theta - 4}$$

$$\sqrt{\ln^2 w - 4} = \sqrt{4(\sec^2 \theta - 1)}$$

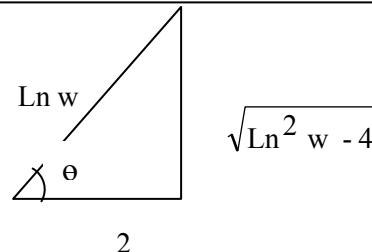
$$\sqrt{\ln^2 w - 4} = \sqrt{4(\tan^2 \theta)}$$

$$\sqrt{\ln^2 w - 4} = 2 \tan \theta$$

$$\text{si } \ln w = 2 \sec \theta \Rightarrow \sec \theta = \frac{\ln w}{2}$$

$$\cos \theta = \frac{2}{\ln w}$$

$$\tan \theta = \frac{\sqrt{\ln^2 w - 4}}{2}$$



$$\frac{1}{3} \left(\ln^2 w - 4 \right) * \left(\sqrt{\ln^2 w - 4} \right) + 4 \left(\sqrt{\ln^2 w - 4} \right) + c =$$

Factorizando términos semejantes

$$\left(\sqrt{\ln^2 w - 4} \right) * \left[\frac{\ln^2 w - 4}{3} + 4 \right] + c$$

Resolviendo términos semejantes

$$\left(\sqrt{\ln^2 w - 4} \right) * \left[\frac{\ln^2 w - 4 + 12}{3} \right] + c = \frac{\left(\sqrt{\ln^2 w - 4} \right) * \left[\ln^2 w - 4 + 12 \right]}{3} + c = \frac{\left(\sqrt{\ln^2 w - 4} \right) * \left[\ln^2 w + 8 \right]}{3} + c$$

$$\int \frac{\ln^3 w \ dw}{w \sqrt{\ln^2 w - 4}} = \frac{\left(\sqrt{\ln^2 w - 4} \right) * \left[\ln^2 w + 8 \right]}{3} + c$$

Ejercicios 7.3 Pag 571 Leythold

Problema # 18

$$\int \frac{dx}{(16+x^2)^{3/2}}$$

$$\int \frac{dx}{(16+x^2)^{3/2}} = \int \frac{4 \sec^2 z dz}{64 \sec^3 z}$$

$$\frac{1}{16} \int \frac{1}{\sec} d\theta = \frac{1}{16} \int \cos \theta d\theta$$

Tabla de integrales

$$\int \cos \theta d\theta = \sin \theta + C$$

$$\frac{1}{16} \int \cos z dz = \frac{1}{16} \sin z + C$$

Reemplazando

$$\frac{1}{16} (\sin z) + C = \frac{1}{16} \left(\frac{x}{\sqrt{16+x^2}} \right) + C$$

$$\int \frac{dx}{(16+x^2)^{3/2}} = \frac{x}{16 \sqrt{16+x^2}} + C$$

$$\begin{aligned} (a^2 + x^2)^{3/2} &\Rightarrow x = a \tan z \\ (16+x^2)^{3/2} &= [(4)^2 + x^2]^{3/2} \Rightarrow x = 4 \tan z \end{aligned}$$

$$x = 4 \tan z$$

$$x^2 = 16 \tan^2 z$$

$$\text{si } x = 4 \tan z \Rightarrow dx = 4 \sec^2 z dz$$

$$(16+x^2)^{3/2} = [16+16 \tan^2 z]^{3/2}$$

$$(16+x^2)^{3/2} = [16(1+\tan^2 z)]^{3/2}$$

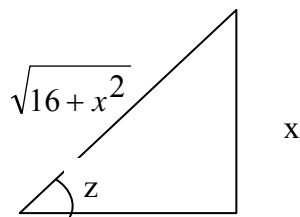
$$(16+x^2)^{3/2} = [16(\sec^2 z)]^{3/2}$$

$$(16+x^2)^{3/2} = [4^2]^{3/2} * [\sec^2 z]^{3/2}$$

$$(16+x^2)^{3/2} = 4^3 \sec^3 z = 64 \sec^3 z$$

$$\text{si } x = 4 \tan z \Rightarrow \tan z = \frac{x}{4}$$

$$\sin z = \frac{x}{\sqrt{16+x^2}}$$



4

Ejercicios 7.3 Pag 571 Leythold
Problema # 20

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

$$\int \frac{dx}{\sqrt{4x - x^2}} = \int \frac{dx}{\sqrt{-\left(x^2 - 4x\right)}}$$

se suma 4 y se resta 4

$$\int \frac{dx}{\sqrt{-\left(x^2 - 4x + 4 - 4\right)}}$$

Ordenando para formar una diferencia de cuadrados

$$\int \frac{dx}{\sqrt{-\left(x^2 - 4x + 4 - 4\right)}} = \int \frac{dx}{\sqrt{-\left(\left[x^2 - 4x + 4\right] - 4\right)}}$$

$$\int \frac{dx}{\sqrt{-\left([x-2]^2 - 4\right)}}$$

$$\int \frac{dx}{\sqrt{4 - (x-2)^2}}$$

reemplazando

$$\int \frac{dx}{\sqrt{4 - (x-2)^2}} = \int \frac{2 \cos z dz}{2 \cos z}$$

$$\int \frac{2 \cos z dz}{2 \cos z} = \int dz$$

$$\int dz = z + c = \arcsen\left(\frac{x-2}{2}\right) + c$$

$$\int \frac{dx}{\sqrt{4x - x^2}} = \arcsen\left(\frac{x-2}{2}\right) + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{4 - (x-2)^2} = \sqrt{2^2 - (x-2)^2}$$

$$\Rightarrow x - 2 = 2 \sin z$$

$$(x-2)^2 = 4 \sin^2 z$$

$$\text{si } x - 2 = 2 \sin z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4 - (x-2)^2} = \sqrt{4 - 4 \sin^2 z}$$

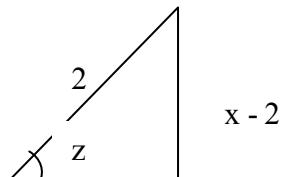
$$\sqrt{4 - (x-2)^2} = \sqrt{4(1 - \sin^2 z)}$$

$$\sqrt{4 - (x-2)^2} = \sqrt{4(\cos^2 z)}$$

$$\sqrt{4 - (x-2)^2} = 2 \cos z$$

$$\text{si } x - 2 = 2 \sin z \Rightarrow \sin z = \frac{x-2}{2}$$

$$z = \arcsen\left(\frac{x-2}{2}\right)$$



$$\sqrt{4 - (x-2)^2}$$

Ejercicios 7.3 Pag 571 Leythold

Problema # 21

$$\int \frac{dx}{(5 - 4x - x^2)^{3/2}}$$

$$\int \frac{dx}{(5 - 4x - x^2)^{3/2}} = \int \frac{dx}{\left[-(x^2 + 4x - 5)\right]^{3/2}}$$

se suma 4 y se resta 4 para formar una diferencia de cuadrados

$$\int \frac{dx}{\left[-(x^2 + 4x + 4 - 5 - 4)\right]^{3/2}} = \int \frac{dx}{\left[-([x+2]^2 - 9)\right]^{3/2}}$$

$$\int \frac{dx}{\left[-[x+2]^2 - 9\right]^{3/2}} = \int \frac{dx}{\left[9 - [x+2]^2\right]^{3/2}}$$

Reemplazando

$$\int \frac{dx}{[9 - [x+2]]^{3/2}} = \int \frac{3 \cos z dz}{27 \cos^3 \theta}$$

$$\frac{1}{9} \int \frac{1}{\cos^2 z} dz$$

$$\frac{1}{9} \int \sec^2 z dz$$

Tabla de integrales

$$\int \sec^2 z dz = \tan z + C$$

$$\frac{1}{9} \int \sec^2 z dz = \frac{1}{9} \tan z + C$$

$$\int \frac{dx}{(5 - 4x - x^2)^{3/2}} = \frac{1}{9} \tan z + C$$

$$\int \frac{dx}{(5 - 4x - x^2)^{3/2}} = \frac{1}{9} \left(\frac{x+2}{\sqrt{9-(x+2)^2}} \right) + C$$

$$(a^2 - x^2)^{3/2} \Rightarrow x = a \sin z$$

$$[3^2 - (x+2)^2]^{3/2} \Rightarrow x+2 = 3 \sin z$$

$$x+2 = 3 \sin z$$

$$(x+2)^2 = 9 \sin^2 z$$

$$\text{si } x+2 = 3 \sin z \Rightarrow dx = 3 \cos z dz$$

$$[9 - (x+2)^2]^{3/2} = [9 - 9 \sin^2 z]^{3/2}$$

$$[9 - (x+2)^2]^{3/2} = [9(1 - \sin^2 z)]^{3/2}$$

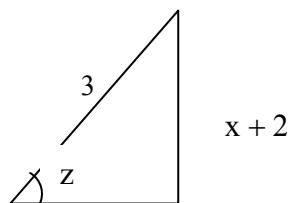
$$[9 - (x+2)^2]^{3/2} = [9(\cos^2 z)]^{3/2}$$

$$[9 - (x+2)^2]^{3/2} = (9)^{3/2} (\cos^2 z)$$

$$[9 - (x+2)^2]^{3/2} = 27 (\cos^3 z)$$

$$\text{si } x+2 = 3 \sin z \Rightarrow \sin z = \frac{x+2}{3}$$

$$\tan z = \frac{x+2}{\sqrt{9 - (x+2)^2}}$$



$$\sqrt{9 - (x+2)^2}$$

Ejercicios 7.3 Pag 571 Leythold
Problema # 22

$$\int \frac{dx}{x \sqrt{x^4 - 4}}$$

$$\int \frac{(x^3) dx}{(x^3)x \sqrt{x^4 - 4}} = \int \frac{x^3 dx}{x^4 \sqrt{x^4 - 4}}$$

Cambio de variable

$$\begin{aligned} u^2 &= x^4 - 4 & 2u du &= 4x^3 dx \\ && \frac{u du}{2} &= x^3 dx \\ u^2 + 4 &= x^4 \end{aligned}$$

$$\begin{aligned} \int \frac{x^3 dx}{x^4 \sqrt{x^4 - 4}} &= \int \frac{\frac{u du}{2}}{(u^2 + 4) * \sqrt{u^2}} = \int \frac{u du}{2(u^2 + 4) * u} \\ \frac{1}{2} \int \frac{du}{u^2 + 4} \end{aligned}$$

$$\frac{1}{2} \int \frac{2 \sec^2 z dz}{4 \sec^2 z}$$

$$\frac{1}{4} \int dz$$

$$\frac{1}{4} \int dz = \frac{1}{4} z + c = \frac{1}{4} \arctan \left(\frac{\sqrt{x^4 - 4}}{2} \right) + c$$

$$\int \frac{dx}{x \sqrt{x^4 - 4}} = \frac{1}{4} \arctan \left(\frac{\sqrt{x^4 - 4}}{2} \right) + c$$

$$u^2 + a^2 \Rightarrow u = a \tan z$$

$$u^2 + 4 = u^2 + 2^2 \Rightarrow u = 2 \tan z$$

$$\begin{aligned} u &= 2 \tan z \\ u^2 &= 4 \tan^2 z \end{aligned}$$

$$\text{si } u = 2 \tan z \Rightarrow du = 2 \sec^2 z dz$$

$$u^2 + 4 = 4 \tan^2 z + 4$$

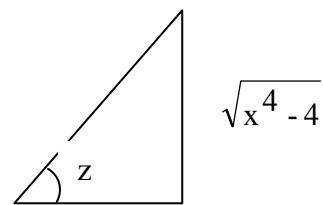
$$u^2 + 4 = 4(\tan^2 z + 1)$$

$$u^2 + 4 = 4(\sec^2 z)$$

$$\text{si } u = 2 \tan z \Rightarrow \tan z = \frac{u}{2} = \frac{\sqrt{x^4 - 4}}{2}$$

$$\text{si } u^2 = x^4 - 4 \Rightarrow u = \sqrt{x^4 - 4}$$

$$z = \arctan \left(\frac{u}{2} \right) = \arctan \left(\frac{\sqrt{x^4 - 4}}{2} \right)$$



Ejercicios 7.3 Pag 571 Leythold

Problema # 23

$$\int \frac{dx}{x^2 \sqrt{x^2 + 9}}$$

$$\int \frac{dx}{x^2 \sqrt{x^2 + 9}} = \int \frac{3 \sec^2 z dz}{(9 \tan^2 z) * (3 \sec z)}$$

$$\frac{1}{9} \int \frac{\sec z}{\tan^2 z} dz$$

$$\frac{1}{9} \int \frac{1}{\frac{\cos z}{\frac{\sin^2 z}{\cos^2 z}}} dz$$

$$\frac{1}{9} \int \frac{\cos z}{\sin^2 z} dz$$

$$u = \sin z \quad du = \cos z \, dz$$

$$u^2 = \sin^2 z$$

Reemplazando

$$\frac{1}{9} \int \frac{\cos z \, dz}{\sin^2 z} = \frac{1}{9} \int \frac{du}{u^2} = \frac{1}{9} \int u^{-2} du$$

$$\frac{1}{9} \int u^{-2} du = \frac{1}{9(-1)} u^{-1} + c = -\frac{1}{9u} + c$$

Reemplazando

$$-\frac{1}{9u} + c = -\frac{1}{9(\sin z)} + c = -\frac{1}{9} * \csc z + c$$

$$-\frac{1}{9} \csc z + c = -\frac{1}{9} \left(\frac{\sqrt{x^2 + 9}}{x} \right) + c = -\frac{1}{9} \frac{\sqrt{x^2 + 9}}{x} + c$$

$$\int \frac{dx}{x^2 \sqrt{x^2 + 9}} = -\frac{1}{9} \frac{\sqrt{x^2 + 9}}{x} + c$$

$$\sqrt{x^2 + a^2} \Rightarrow x = a \tan z$$

$$\sqrt{x^2 + 9} = \sqrt{x^2 + 3^2} \Rightarrow x = 3 \tan z$$

$$x = 3 \tan z$$

$$x^2 = 9 \tan^2 z$$

$$\text{si } x = 3 \tan z \Rightarrow dx = 3 \sec^2 z \, dz$$

$$\sqrt{x^2 + 9} = \sqrt{9 \tan^2 z + 9}$$

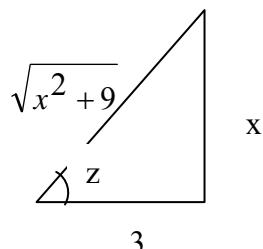
$$\sqrt{x^2 + 9} = \sqrt{9 (\tan^2 z + 1)}$$

$$\sqrt{x^2 + 9} = \sqrt{9 (\sec^2 z)}$$

$$\sqrt{x^2 + 9} = 3 \sec z$$

$$\text{si } x = 3 \tan z \Rightarrow \tan z = \frac{x}{3}$$

$$\sin z = \frac{x}{\sqrt{x^2 + 9}} \quad \csc z = \frac{\sqrt{x^2 + 9}}{x}$$



Ejercicios 7.3 Pag 571 Leythold

Problema # 24

$$\int \frac{x^2}{\sqrt{4-x^2}} dx$$

$$\int \frac{x^2 dx}{\sqrt{4-x^2}} = \int \frac{4 \sin^2 z (2 \cos z dz)}{(2 \cos z)}$$

$$\int 4 \sin^2 z dz = 4 \int \sin^2 z dz$$

Identidades trigonometricas

$$\frac{1-\cos 2z}{2} = \sin^2 z$$

$$4 \int \sin^2 z dz = 4 \int \left(\frac{1-\cos 2z}{2} \right) dz$$

$$\frac{4}{2} \int (1 - \cos 2z) dz = [2(\int 1 dz - \int \cos 2z dz)]$$

$$2 \int dz - 2 \int \cos 2z dz$$

$$2z - 2 \left(\frac{1}{2} \sin 2z \right) + c$$

Identidades trigonometricas

$$\sin 2z = 2 \sin z * \cos z$$

$$2z - 2 \left(\frac{1}{2} \sin 2z \right) + c = 2z - (\sin z * \cos z) + c$$

Reemplazando

$$2z - (\sin z * \cos z) + c = 2 \left(\arcsen \left(\frac{x}{2} \right) \right) - \left[\frac{x}{2} * \left(\frac{\sqrt{4-x^2}}{2} \right) \right]$$

$$2 \left(\arcsen \left(\frac{x}{2} \right) \right) - \left[\frac{x}{2} * \left(\frac{\sqrt{4-x^2}}{2} \right) \right] + c = 2 \arcsen \left(\frac{x}{2} \right) - \left(\frac{x \sqrt{4-x^2}}{4} \right) + c$$

$$2 \arcsen \left(\frac{x}{2} \right) - \left(\frac{x \sqrt{4-x^2}}{4} \right) + c = 2 \arcsen \left(\frac{x}{2} \right) - \frac{1}{4} \left(x \sqrt{4-x^2} \right) + c$$

$$\int \frac{x^2}{\sqrt{4-x^2}} dx = 2 \arcsen \left(\frac{x}{2} \right) - \frac{1}{4} \left(x \sqrt{4-x^2} \right) + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{4-x^2} = \sqrt{2^2 - x^2}$$

$$\sqrt{2^2 - x^2} \Rightarrow x = 2 \sin z$$

$$x = 2 \sin z$$

$$x^2 = 4 \sin^2 z$$

$$\text{si } x = 2 \sin z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4-x^2} = \sqrt{4 - 4 \sin^2 z}$$

$$\sqrt{4-x^2} = \sqrt{4 (1 - \sin^2 z)}$$

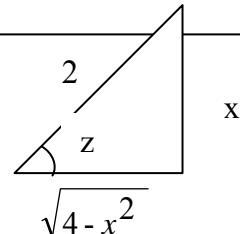
$$\sqrt{4-x^2} = \sqrt{4 (\cos^2 z)}$$

$$\sqrt{4-x^2} = 2 \cos z$$

$$\text{si } x = 2 \sin z \Rightarrow \sin z = \frac{x}{2}$$

$$z = \arcsen \left(\frac{x}{2} \right)$$

$$\text{si } \sqrt{4-x^2} = 2 \cos z \Rightarrow \cos z = \frac{\sqrt{4-x^2}}{2}$$



Ejercicios 7.3 Pag 571 Leythold
Problema # 25

$$\int \frac{dx}{x\sqrt{x^2 - 4}}$$

$$\int \frac{dx}{x\sqrt{x^2 - 4}} = \int \frac{2 \sec z \tan z \ dz}{(2 \sec z)(2 \tan z)}$$

$$\frac{1}{2} \int dz$$

Tabla de integrales

$$\int dz = z + c$$

$$\int \frac{dx}{x\sqrt{x^2 - 4}} = \frac{1}{2} \int dz$$

$$\frac{1}{2}(z) + c$$

$$\int \frac{dx}{x\sqrt{x^2 - 4}} = \frac{1}{2} \left(\operatorname{arcsec} \frac{x}{2} \right) + c$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 4} = \sqrt{x^2 - 2^2}$$

$$\sqrt{x^2 - 2^2} \Rightarrow x = 2 \sec z$$

$$x = 2 \sec z$$

$$x^2 = 4 \sec^2 z$$

$$\text{si } x = 2 \sec z \Rightarrow dx = 2 \sec z \tan z \ dz$$

$$\sqrt{x^2 - 4} = \sqrt{4 \sec^2 z - 4}$$

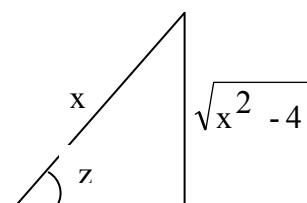
$$\sqrt{x^2 - 4} = \sqrt{4(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 4} = \sqrt{4(\tan^2 z)}$$

$$\sqrt{x^2 - 4} = 2 \tan z$$

$$x = 2 \sec z \Rightarrow \sec z = \frac{x}{2}$$

$$z = \operatorname{arcsec} \frac{x}{2}$$



2

Ejercicios 7.3 Pag 571 Leythold
Problema # 28

$$\int \frac{dw}{(w^2 - 4)^{3/2}}$$

$$\int \frac{dw}{(w^2 - 4)^{3/2}} = \int \frac{2 \sec z \tan z \ dz}{(8 \tan^3 z)}$$

$$\frac{1}{4} \int \frac{\sec z}{\tan^2 z} dz$$

$$\frac{1}{4} \int \frac{(\sec z)}{\tan^2 z} dz = \frac{1}{4} \int \frac{1}{\frac{\sin^2 z}{\cos^2 z}} dz = \frac{1}{4} \int \frac{\cos z}{\sin^2 z} dz$$

$$\frac{1}{4} \int \frac{\cos z}{\sin z} * \frac{1}{\sin z} dz = \frac{1}{4} \int \cot z * \csc z dz$$

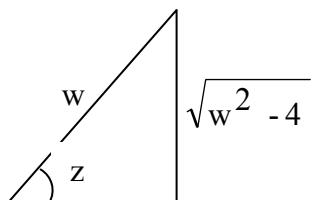
Tabla de integrales

$$\int \csc z \cot z dz = -\csc z + c$$

$$\frac{1}{4} \int \cot z * \csc z dz = -\frac{1}{4} \csc z + c$$

$$\int \frac{dw}{(w^2 - 4)^{3/2}} = -\frac{1}{4} \csc z + c = -\frac{1}{4} \left(\frac{w}{\sqrt{w^2 - 4}} \right) + c$$

$$\int \frac{dw}{(w^2 - 4)^{3/2}} = -\frac{1}{4} \left(\frac{w}{\sqrt{w^2 - 4}} \right) + c$$



2

$$(w^2 - a^2)^{3/2} \Rightarrow x = a \sec z$$

$$(w^2 - 4)^{3/2} = (w^2 - 2^2)^{3/2}$$

$$(w^2 - 2^2)^{3/2} \Rightarrow w = 2 \sec z$$

$$w = 2 \sec z$$

$$w^2 = 4 \sec^2 z$$

$$\text{si } w = 2 \sec z \Rightarrow dw = 2 \sec z \tan z dz$$

$$(w^2 - 4)^{3/2} = (4 \sec^2 z - 4)^{3/2}$$

$$(w^2 - 4)^{3/2} = (4(\sec^2 z - 1))^{3/2}$$

$$(w^2 - 4)^{3/2} = (4(\tan^2 z))^{3/2}$$

$$(w^2 - 4)^{3/2} = (2^2)^{3/2} \tan^3 z$$

$$(w^2 - 4)^{3/2} = 2^3 \tan^3 z = 8 \tan^3 z$$

$$w = 2 \sec z \Rightarrow \sec z = \frac{w}{2}$$

$$z = \arccos \frac{w}{2}$$

$$\sin z = \frac{\sqrt{w^2 - 4}}{w} \Rightarrow \csc z = \frac{w}{\sqrt{w^2 - 4}}$$

Ejercicios 7.3 Pag 571 Leythold

Problema # 29

$$\int \frac{e^t dt}{\left(e^{2t} + 8e^t + 7\right)^{3/2}} = \int \frac{e^t dt}{\left(e^{2t} + 8e^t + 16 + 7 - 16\right)^{3/2}}$$

$$\int \frac{e^t dt}{\left[\left(e^{2t} + 8e^t + 16\right) + 7 - 16\right]^{3/2}}$$

$$\int \frac{e^t dt}{\left[\left(e^t + 4\right)^2 + 7 - 16\right]^{3/2}}$$

$$\int \frac{e^t dt}{\left[\left(e^t + 4\right)^2 - 9\right]^{3/2}}$$

Reemplazando

$$\int \frac{e^t dt}{\left[\left(e^t + 4\right)^2 - 9\right]^{3/2}} = \int \frac{3 \sec \theta \operatorname{tg} \theta d\theta}{27 \operatorname{tg}^3 \theta} = \frac{1}{9} \int \sec \theta \frac{1}{\operatorname{tg}^2 \theta} d\theta$$

$$\frac{1}{9} \int \frac{1}{\cos \theta} * \operatorname{ctg}^2 \theta d\theta = \frac{1}{9} \int \frac{1}{\cos \theta} * \frac{\cos^2 \theta}{\sin^2 \theta} d\theta$$

$$\frac{1}{9} \int \frac{\cos \theta}{\sin^2 \theta} d\theta = \frac{1}{9} \int \frac{\cos \theta}{\sin \theta} * \frac{1}{\sin \theta} d\theta$$

$$\frac{1}{9} \int \cot \theta * \csc \theta d\theta =$$

Tabla de integrales

$$\int \csc \theta \operatorname{ctg} \theta d\theta = -\csc \theta + C$$

$$\frac{1}{9} \int \cot \theta * \csc \theta d\theta = -\frac{1}{9} \csc \theta + C = -\frac{1}{9} \left(\frac{e^t + 4}{\sqrt{\left(e^t + 4\right)^2 - 9}} \right)$$

$$\left(e^t + 4\right)^2 - 9^2 \Rightarrow \left(e^t + 4\right) = 3 \sec \theta$$

$$e^t + 4 = 3 \sec \theta$$

$$\left(e^t + 4\right)^2 = 9 \sec^2 \theta$$

$$\text{si } e^t + 4 = 3 \sec \theta \Rightarrow e^t dt = 3 \sec \theta \operatorname{tg} \theta d\theta$$

$$\left[\left(e^t + 4\right)^2 - 9\right]^{\frac{3}{2}} = \left[9 \sec^2 \theta - 9\right]^{\frac{3}{2}}$$

$$\left[\left(e^t + 4\right)^2 - 9\right]^{\frac{3}{2}} = \left[9(\sec^2 \theta - 1)\right]^{\frac{3}{2}}$$

$$\left[\left(e^t + 4\right)^2 - 9\right]^{\frac{3}{2}} = \left[9(\operatorname{tg}^2 \theta)\right]^{\frac{3}{2}}$$

$$\left[\left(e^t + 4\right)^2 - 9\right]^{\frac{3}{2}} = [9]^{3/2} \operatorname{tg}^3 \theta$$

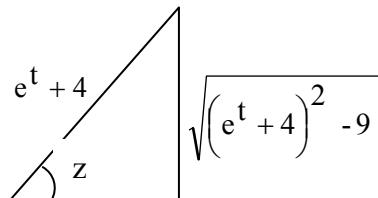
$$\left[\left(e^t + 4\right)^2 - 9\right]^{\frac{3}{2}} = 27 \operatorname{tg}^3 \theta$$

$$\left[(z-3)^2 + 9\right]^{\frac{3}{2}} = 27 \sec^3 \theta$$

$$\text{si } e^t + 4 = 3 \sec \theta \Rightarrow \sec \theta = \frac{e^t + 4}{3}$$

$$\operatorname{sen} \theta = \frac{\sqrt{\left(e^t + 4\right)^2 - 9}}{e^t + 4}$$

$$\csc \theta = \frac{e^t + 4}{\sqrt{\left(e^t + 4\right)^2 - 9}}$$



Ejercicios 7.3 Pag 571 Leythold
Problema # 30

$$\int \frac{\sqrt{16 - e^{2x}}}{e^x} dx$$

$$\int \frac{\sqrt{16 - e^{2x}}}{e^x} dx = \int \frac{e^x * \sqrt{16 - e^{2x}}}{e^x * (e^x)} dx$$

$$\int \frac{e^x * \sqrt{16 - e^{2x}}}{e^x * (e^x)} dx = \int \frac{\sqrt{16 - e^{2x}} (e^x dx)}{e^{2x}}$$

$$\int \frac{\sqrt{16 - e^{2x}} (e^x dx)}{e^{2x}} = \int \frac{(4 \cos z)(4 \cos z dz)}{16^2 \sin^2 z}$$

$$\int \frac{(4 \cos z)(4 \cos z dz)}{16^2 \sin^2 z} = \int \frac{\cos^2 z dz}{\sin^2 z} = \int \operatorname{ctg}^2 z dz$$

Identidades trigonométricas

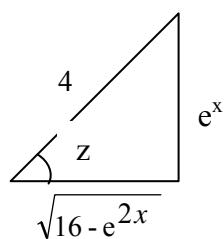
$$\cot^2 z = \csc^2 z - 1$$

$$\int \operatorname{ctg}^2 z dz = \int (\csc^2 z - 1) dz = \int \csc^2 z dz - \int dz$$

$$\int \csc^2 z dz - \int dz = -\operatorname{ctg} z - z + c$$

$$\int \frac{\sqrt{16 - e^{2x}}}{e^x} dx = -\operatorname{ctg} z - z + c = -\left(\frac{\sqrt{16 - e^{2x}}}{e^x} \right) - \operatorname{arc sen} \left(\frac{e^x}{4} \right) + c$$

$$\int \frac{\sqrt{16 - e^{2x}}}{e^x} dx = -\left(\frac{\sqrt{16 - e^{2x}}}{e^x} \right) - \operatorname{arc sen} \left(\frac{e^x}{4} \right) + c$$



$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{16 - e^{2x}} = \sqrt{4^2 - (e^x)^2}$$

$$\sqrt{4^2 - (e^x)^2} \Rightarrow e^x = 4 \sin z$$

$$e^x = 4 \sin z$$

$$e^{2x} = 16 \sin^2 z$$

$$\text{si } e^x = 4 \sin z \Rightarrow e^x dx = 4 \cos z dz$$

$$\sqrt{16 - e^{2x}} = \sqrt{16 - 16 \sin^2 z}$$

$$\sqrt{16 - e^{2x}} = \sqrt{16(1 - \sin^2 z)}$$

$$\sqrt{16 - e^{2x}} = \sqrt{16(\cos^2 z)}$$

$$\sqrt{16 - e^{2x}} = 4 \cos z$$

$$\text{si } e^x = 4 \sin z \Rightarrow \sin z = \frac{e^x}{4}$$

$$z = \operatorname{arc sen} \left(\frac{e^x}{4} \right)$$

$$\text{si } \sqrt{16 - e^{2x}} = 4 \cos z \Rightarrow \cos z = \frac{\sqrt{16 - e^{2x}}}{4}$$

$$\operatorname{ctg} z = \frac{\sqrt{16 - e^{2x}}}{e^x}$$

Ejercicio # 2 Pag 270 Calculo diferencial e integral SCHAUM

$$\int \frac{x^2 dx}{\sqrt{x^2 - 4}}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 - 4}} = \int \frac{(4 \sec^2 z) 2 \sec z \tan z dz}{(2 \tan z)}$$

$$4 \int \sec^3 z dz =$$

Se resuelve la integral $4 \int \sec^3 z dz$ **por partes**

$$\int u dv = u * v - \int v du$$

$$4 \int \sec^3 z dz = 4 \int \sec^2 z * \sec z dz$$

Se resuelve por partes

$u = \sec z$	$dv = \sec^2 z$
$du = \sec z \tan z dz$	$\int dv = \int \sec^2 z dz$
	$v = \tan z$

$$4 \int \sec^2 z * \sec z dz = 4 [\sec z * \tan z - \int (\tan z)^2 * \sec z dz]$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 4} = \sqrt{x^2 - 2^2}$$

$$\sqrt{x^2 - 2^2} \Rightarrow x = 2 \sec z$$

$$x = 2 \sec z$$

$$x^2 = 4 \sec^2 z$$

$$\text{si } x = 2 \sec z \Rightarrow dx = 2 \sec z \tan z dz$$

$$\sqrt{x^2 - 4} = \sqrt{4 \sec^2 z - 4}$$

$$\sqrt{x^2 - 4} = \sqrt{4(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 4} = \sqrt{4(\tan^2 z)}$$

$$\sqrt{x^2 - 4} = 2 \tan z$$

$$4 [\sec z * \tan z - \int (\tan z)^2 * \sec z dz] = 4 [\sec z * \tan z - \int \tan^2 z * \sec z dz]$$

Reemplazando la Identidad trigonométrica $\tan^2 z = \sec^2 z - 1$

$$4 [\sec z * \tan z - \int \tan^2 z * \sec z dz] = 4 [\sec z * \tan z - \int (\sec^2 z - 1) * \sec z dz]$$

$$4 [\sec z * \tan z - \int \sec^3 z dz + \int \sec z dz]$$

$$4 \int \sec^3 z dz = 4 [\sec z * \tan z - \int \sec^3 z dz + \int \sec z dz]$$

$$4 \int \sec^3 z dz = 4 \sec z * \tan z - 4 \int \sec^3 z dz + 4 \int \sec z dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$4 \int \sec^3 z dz + 4 \int \sec^3 z dz = 4 \sec z * \tan z + 4 \int \sec z dz$$

$$8 \int \sec^3 z dz = 4 \sec z * \tan z + 4 \int \sec z dz$$

Dividiendo la ecuación por 2

$$4 \int \sec^3 z dz = 2 \sec z * \tan z + 2 \int \sec z dz$$

Tabla de integrales

$$\int \sec z \, dz = \ln |\sec z + \tan z| + c$$

$$4 \int \sec^3 z \, dz = 2 \sec z * \tan z + 2 \ln |\sec z + \tan z| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 4}} = 2 \sec z * \tan z + 2 \ln |\sec z + \tan z| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 4}} = 2 \left(\frac{x}{2} \right) * \left(\frac{\sqrt{x^2 - 4}}{2} \right) + 2 \ln \left| \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2} \right| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 4}} = \left(\frac{x \sqrt{x^2 - 4}}{2} \right) + 2 \ln \left| \frac{x + \sqrt{x^2 - 4}}{2} \right| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 4}} = \left(\frac{x \sqrt{x^2 - 4}}{2} \right) + 2 \ln \left| x + \sqrt{x^2 - 4} \right| - 2 \ln |2| + c$$

Pero: $C_1 = -2 \ln |4| + c$

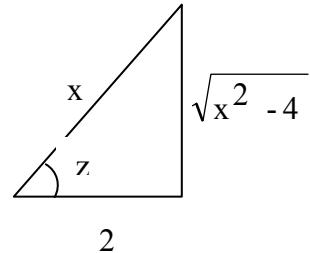
$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 4}} = \left(\frac{x \sqrt{x^2 - 4}}{2} \right) + 2 \ln \left| x + \sqrt{x^2 - 4} \right| + c$$

$$x = 2 \sec z$$

$$\text{si } x = 2 \sec z \Rightarrow \sec z = \frac{x}{2}$$

$$\cos z = \frac{2}{x}$$

$$\tan z = \frac{\sqrt{x^2 - 4}}{2}$$



Ejercicio # 3 Pag 270
Calculo diferencial e integral SCHAUM

$$\int \frac{\sqrt{9 - 4x^2} dx}{x} = \int \frac{\sqrt{9 - 4x^2}}{x} dz = \int \frac{3 \cos z \left(\frac{3}{2} \cos z dz \right)}{\frac{3}{2} \sin z}$$

Identidad trigonométrica
 $\cos^2 z = 1 - \sin^2 z$

$$3 \int \frac{\cos^2 z}{\sin z} dz = 3 \int \frac{(1 - \sin^2 z)}{\sin z} dz$$

$$\int \frac{3 \cos z \left(\frac{3}{2} \cos z dz \right)}{\frac{3}{2} \sin z} = 3 \int \frac{\cos^2 z}{\sin z} dz$$

$$3 \int \frac{(1 - \sin^2 z)}{\sin z} dz = 3 \int \left[\frac{1}{\sin z} - \frac{\sin^2 z}{\sin z} \right] dz$$

$$3 \int [\csc z - \cot z] dz$$

$$3 \int \csc z dz - 3 \int \cot z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \cot z| + c$$

$$\int \cot z dz = -\ln |\sin z| + c$$

$$3 \int \csc z dz - 3 \int \cot z dz = 3 \ln |\csc z - \cot z| - 3(-\ln |\sin z|) + c$$

$$3 \int \csc z dz - 3 \int \cot z dz = 3 \ln |\csc z - \cot z| + 3 \ln |\sin z| + c$$

$$\int \frac{\sqrt{9 - 4x^2} dx}{x} = 3 \ln |\csc z - \cot z| + 3 \ln |\sin z| + c$$

$$\int \frac{\sqrt{9 - 4x^2} dx}{x} = 3 \ln \left| \frac{3}{2x} - \frac{\sqrt{9 - 4x^2}}{2x} \right| + 3 \left(\frac{\sqrt{9 - 4x^2}}{3} \right) + c$$

$$\int \frac{\sqrt{9 - 4x^2} dx}{x} = 3 \ln \left| \frac{3 - \sqrt{9 - 4x^2}}{2x} \right| + \sqrt{9 - 4x^2} + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{9 - 4x^2} = \sqrt{\frac{9}{4} - \frac{4x^2}{4}} = \sqrt{\left(\frac{3}{2}\right)^2 - x^2}$$

$$\sqrt{9 - 4x^2} = \sqrt{\left(\frac{3}{2}\right)^2 - x^2} \Rightarrow x = \frac{3}{2} \sin z$$

$$x = \frac{3}{2} \sin z \quad x^2 = \frac{9}{4} \sin^2 z$$

$$\text{si } x = \frac{3}{2} \sin z \Rightarrow dx = \frac{3}{2} \cos z dz$$

$$\sqrt{9 - 4x^2} = \sqrt{9 - 4\left(\frac{9}{4} \sin^2 z\right)}$$

$$\sqrt{9 - 4x^2} = \sqrt{9 - 9 \sin^2 z}$$

$$\sqrt{9 - 4x^2} = \sqrt{9(1 - \sin^2 z)}$$

$$\sqrt{9 - 4x^2} = \sqrt{9(\cos^2 z)}$$

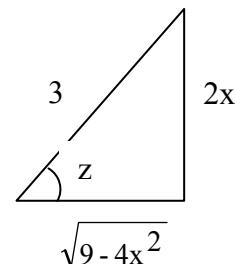
$$\sqrt{9 - 4x^2} = 3 \cos z$$

$$\text{si } x = \frac{3}{2} \sin z \Rightarrow \sin z = \frac{2x}{3}$$

$$\text{si } \sqrt{9 - 4x^2} = 3 \cos z \Rightarrow \cos z = \frac{\sqrt{9 - 4x^2}}{3}$$

$$\tan z = \frac{2x}{\sqrt{9 - 4x^2}} \quad \cot z = \frac{\sqrt{9 - 4x^2}}{2x}$$

$$\csc z = \frac{3}{2x}$$



Ejercicio # 4 Pag 270 Calculo diferencial e integral SCHAUM

$$\int \frac{dx}{x \sqrt{9+4x^2}}$$

$$\int \frac{dx}{x \sqrt{9+4x^2}} = \int \frac{\frac{3}{2} \sec^2 z dz}{\left(\frac{3}{2} \operatorname{tg} z\right)^* (3 \sec z)}$$

$$\int \frac{1}{3} \frac{\sec z}{\operatorname{tg} z} dz$$

$$\frac{1}{3} \int \frac{\cos z}{\sin z} dz$$

$$\frac{1}{3} \int \frac{1}{\sin z} dz$$

$$\frac{1}{3} \int \csc z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \cot z| + c$$

$$\int \frac{dx}{x \sqrt{9+4x^2}} = \frac{1}{3} \int \csc z dz$$

$$\int \frac{dx}{x \sqrt{9+4x^2}} = \frac{1}{3} \ln |\csc z - \cot z| + c$$

Reemplazando

$$\int \frac{dx}{x \sqrt{9+4x^2}} = \frac{1}{3} \ln \left| \frac{\sqrt{9+4x^2}}{2x} - \frac{3}{2x} \right| + c$$

$$\int \frac{dx}{x \sqrt{9+4x^2}} = \frac{1}{3} \ln \left| \frac{\sqrt{9+4x^2} - 3}{2x} \right| + c$$

$$\sqrt{a^2 + x^2} \Rightarrow x = a \operatorname{tg} z$$

$$\sqrt{9+4x^2} = \sqrt{\frac{9}{4} + \frac{4x^2}{4}}$$

$$\sqrt{9+4x^2} = \sqrt{\left(\frac{3}{2}\right)^2 + x^2} \Rightarrow x = \frac{3}{2} \operatorname{tg} z$$

$$x = \frac{3}{2} \operatorname{tg} z$$

$$x^2 = \frac{9}{4} \operatorname{tg}^2 z$$

$$\text{si } x = \frac{3}{2} \operatorname{tg} z \Rightarrow dx = \frac{3}{2} \sec^2 z dz$$

$$\sqrt{9+4x^2} = \sqrt{9 + 4\left(\frac{9}{4} \operatorname{tg}^2 z\right)}$$

$$\sqrt{9+4x^2} = \sqrt{9 + 9 \operatorname{tg}^2 z}$$

$$\sqrt{9+4x^2} = \sqrt{9(1 + \operatorname{tg}^2 z)}$$

$$\sqrt{9+4x^2} = \sqrt{9(\sec^2 z)}$$

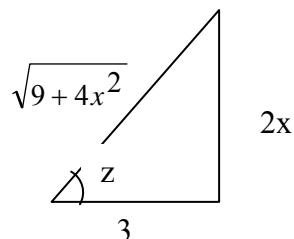
$$\sqrt{9+4x^2} = 3 \sec z$$

$$\text{si } x = \frac{3}{2} \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{2x}{3}$$

$$\text{si } \sqrt{9+4x^2} = 3 \sec z \Rightarrow \sec z = \frac{\sqrt{9+4x^2}}{3}$$

$$\text{si } \operatorname{tg} z = \frac{2x}{3} \Rightarrow \operatorname{ctg} z = \frac{3}{2x}$$

$$\csc z = \frac{\sqrt{9+4x^2}}{2x}$$



Resolver

$$\int \frac{dx}{\sqrt{1 - 4x^2}}$$

$$\int \frac{dx}{\sqrt{1 - 4x^2}} = \int \frac{\frac{1}{2} \cos z dz}{\frac{1}{2} \cos z}$$

$$\int dz$$

Tabla de integrales

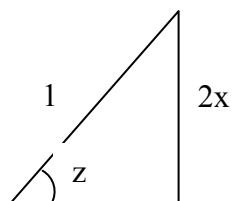
$$\int dz = z + c$$

$$\int \frac{dx}{\sqrt{1 - 4x^2}} = \int dz$$

$$\int \frac{dx}{\sqrt{1 - 4x^2}} = z + c$$

Reemplazando

$$\int \frac{dx}{\sqrt{1 - 4x^2}} = \arcsen(2x) + c$$



$$\sqrt{1 - 4x^2}$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sen z$$

$$\sqrt{1 - 4x^2} = \sqrt{\left(\frac{1}{4}\right) - \left(\frac{4x^2}{4}\right)}$$

$$\sqrt{1 - 4x^2} = \sqrt{\left(\frac{1}{2}\right)^2 - x^2} \Rightarrow x = \frac{1}{2} \sen z$$

$$x = \frac{1}{2} \sen z$$

$$x^2 = \frac{1}{4} \sen^2 z$$

$$\text{si } x = \frac{1}{2} \sen z \Rightarrow dx = \frac{1}{2} \cos z dz$$

$$\sqrt{\frac{1}{4} - x^2} = \sqrt{\frac{1}{4} - \left(\frac{1}{4} \sen^2 z\right)}$$

$$\sqrt{\frac{1}{4} - x^2} = \sqrt{\frac{1}{4} (1 - \sen^2 z)}$$

$$\sqrt{\frac{1}{4} - x^2} = \sqrt{\frac{1}{4} (\cos^2 z)}$$

$$\sqrt{\frac{1}{4} - x^2} = \frac{1}{2} \cos z$$

$$\text{si } x = \frac{1}{2} \sen z$$

$$z = \arcsen(2x)$$

Resolver

$$\int \frac{dx}{4x\sqrt{x^2 - 16}}$$

$$\int \frac{dx}{4x\sqrt{x^2 - 16}} = \int \frac{4 \sec z \tan z \ dz}{4(4 \sec z)(4 \tan z)}$$

$$\frac{1}{16} \int dz$$

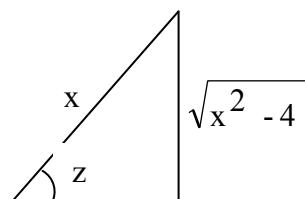
Tabla de integrales

$$\int dz = z + c$$

$$\int \frac{dx}{4x\sqrt{x^2 - 16}} = \frac{1}{16} \int dz$$

$$\int \frac{dx}{4x\sqrt{x^2 - 16}} = \frac{1}{16} (z) + c$$

$$\int \frac{dx}{4x\sqrt{x^2 - 16}} = \frac{1}{16} \left(\operatorname{arcsec} \frac{x}{4} \right) + c$$



$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 16} = \sqrt{x^2 - 4^2}$$

$$\sqrt{x^2 - 4^2} \Rightarrow x = 4 \sec z$$

$$x = 4 \sec z$$

$$x^2 = 16 \sec^2 z$$

$$\text{si } x = 4 \sec z \Rightarrow dx = 4 \sec z \tan z \ dz$$

$$\sqrt{x^2 - 16} = \sqrt{16 \sec^2 z - 16}$$

$$\sqrt{x^2 - 16} = \sqrt{16(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 16} = \sqrt{16(\tan^2 z)}$$

$$\sqrt{x^2 - 16} = 4 \tan z$$

$$x = 4 \sec z \Rightarrow \sec z = \frac{x}{4}$$

$$z = \operatorname{arcsec} \frac{x}{4}$$

Resolver

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx$$

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx = \int \frac{3 \cos z (3 \cos z dz)}{9 \sin^2 z}$$

$$\int \frac{\cos^2 z}{\sin^2 z} dz$$

$$\int \operatorname{ctg}^2 z dz$$

Identidad trigonométrica
 $\operatorname{ctg}^2 z = \csc^2 z - 1$

$$\int \operatorname{ctg}^2 z dz = \int (\csc^2 z - 1) dz$$

$$\int (\csc^2 z - 1) dz = \int \csc^2 z dz - \int 1 dz$$

Tabla de integrales

$$\int dz = z + c$$

$$\int \csc^2 z dz = -\operatorname{ctg} z + c$$

$$\int \csc^2 z dz - \int 1 dz = -\operatorname{ctg} z - z + c$$

Reemplazando

$$-\operatorname{ctg} z - z + c = -\left(\frac{\sqrt{9 - x^2}}{x}\right) - \operatorname{arc sen}\left(\frac{x}{3}\right) + c$$

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx = -\left[\frac{\sqrt{9 - x^2}}{x}\right] - \operatorname{arc sen}\left[\frac{x}{3}\right] + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \operatorname{sen} z$$

$$\sqrt{9 - x^2} = \sqrt{3^2 - x^2}$$

$$\sqrt{3^2 - x^2} \Rightarrow x = 3 \operatorname{sen} z$$

$$x = 3 \operatorname{sen} z$$

$$x^2 = 9 \operatorname{sen}^2 z$$

$$\text{si } x = 3 \operatorname{sen} z \Rightarrow dx = 3 \operatorname{cos} z dz$$

$$\sqrt{9 - x^2} = \sqrt{9 - 9 \operatorname{sen}^2 z}$$

$$\sqrt{9 - x^2} = \sqrt{9(1 - \operatorname{sen}^2 z)}$$

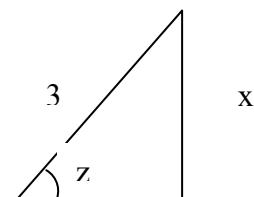
$$\sqrt{9 - x^2} = \sqrt{9(\cos^2 z)}$$

$$\sqrt{9 - x^2} = 3 \cos z$$

$$\text{si } x = 3 \operatorname{sen} z \Rightarrow \operatorname{sen} z = \frac{x}{3}$$

$$\operatorname{sen} z = \frac{x}{3} \Rightarrow z = \operatorname{arc sen}\left(\frac{x}{3}\right)$$

$$\operatorname{tg} z = \frac{x}{\sqrt{9 - x^2}} \quad \operatorname{ctg} z = \frac{\sqrt{9 - x^2}}{x}$$



$$\sqrt{9 - x^2}$$

Resolver

$$\int \sqrt{x^2 + 5} \, dx$$

$$\int \sqrt{x^2 + 5} \, dx = \int \sqrt{5} \sec z (\sqrt{5} \sec^2 z \, dz)$$

$$\int 5 \sec^3 z \, dz$$

Se resuelve la integral $5 \int \sec^3 z \, dz$ **por partes**

$$\int u \, dv = u * v - \int v \, du$$

$$5 \int \sec^3 z \, dz = 5 \int \sec^2 z * \sec z \, dz$$

Se resuelve por partes

$$u = \sec z$$

$$dv = \sec^2 z$$

$$du = \sec z \, \tan z \, dz$$

$$\int dv = \int \sec^2 z \, dz$$

$$v = \tan z$$

$$5 \int \sec^2 z * \sec z \, dz = 5 [\sec z * \tan z - \int (\tan z) * \sec z * \sec z \, dz]$$

$$5 [\sec z * \tan z - \int (\tan z) * \sec z * \sec z \, dz] = 5 [\sec z * \tan z - \int \sec^2 z * \sec z \, dz]$$

Reemplazando la Identidad trigonométrica

$$\tan^2 z = \sec^2 z - 1$$

$$5 [\sec z * \tan z - \int \sec^2 z * \sec z \, dz] = 5 [\sec z * \tan z - \int (\sec^2 z - 1) * \sec z \, dz]$$

$$5 [\sec z * \tan z - \int \sec^3 z \, dz + \int \sec z \, dz]$$

$$5 \int \sec^3 z \, dz = 5 [\sec z * \tan z - \int \sec^3 z \, dz + \int \sec z \, dz]$$

$$5 \int \sec^3 z \, dz = 5 \sec z * \tan z - 5 \int \sec^3 z \, dz + 5 \int \sec z \, dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$5 \int \sec^3 z \, dz + 5 \int \sec^3 z \, dz = 5 \sec z * \tan z + 5 \int \sec z \, dz$$

$$10 \int \sec^3 z \, dz = 5 \sec z * \tan z + 5 \int \sec z \, dz$$

Dividiendo la ecuación por 2

$$5 \int \sec^3 z \, dz = \frac{5}{2} \sec z * \tan z + \frac{5}{2} \int \sec z \, dz$$

$$\sqrt{x^2 + a^2} \Rightarrow x = a \tan z$$

$$\sqrt{x^2 + 5} = \sqrt{x^2 + (\sqrt{5})^2} \Rightarrow x = \sqrt{5} \tan z$$

$$x = \sqrt{5} \tan z$$

$$x^2 = 5 \tan^2 z$$

$$\text{si } x = \sqrt{5} \tan z \Rightarrow dx = \sqrt{5} \sec^2 z \, dz$$

$$\sqrt{x^2 + 5} = \sqrt{5 \tan^2 z + 5}$$

$$\sqrt{x^2 + 5} = \sqrt{5(\tan^2 z + 1)}$$

$$\sqrt{x^2 + 5} = \sqrt{5(\sec^2 z)}$$

$$\sqrt{x^2 + 5} = \sqrt{5} \sec z$$

Tabla de integrales

$$\int \sec z \, dz = \ln |\sec z + \tan z| + c$$

$$5 \int \sec^3 z \, dz = \frac{5}{2} \sec z * \tan z + \frac{5}{2} \ln |\sec z + \tan z| + c$$

$$\int \sqrt{x^2 + 5} \, dx = \frac{5}{2} \sec z * \tan z + \frac{5}{2} \ln |\sec z + \tan z| + c$$

$$\int \sqrt{x^2 + 5} \, dx = \frac{5}{2} \left(\frac{\sqrt{x^2 + 5}}{\sqrt{5}} \right) * \left(\frac{x}{\sqrt{5}} \right) + \frac{5}{2} \ln \left| \frac{\sqrt{x^2 + 5}}{\sqrt{5}} + \frac{x}{\sqrt{5}} \right| + c$$

$$\int \sqrt{x^2 + 5} \, dx = \frac{5}{2} \left(\frac{x \sqrt{x^2 + 5}}{5} \right) + \frac{5}{2} \ln \left| \frac{\sqrt{x^2 + 5} + x}{\sqrt{5}} \right| + c$$

$$\int \sqrt{x^2 + 5} \, dx = \frac{1}{2} \left(x \sqrt{x^2 + 5} \right) + \frac{5}{2} \ln \left| \frac{\sqrt{x^2 + 5} + x}{\sqrt{5}} \right| + c$$

$$x = \sqrt{5} \tan z$$

$$\text{si } x = \sqrt{5} \tan z \Rightarrow \tan z = \frac{x}{\sqrt{5}}$$

$$\cos z = \frac{2}{x}$$

$$\tan z = \frac{\sqrt{x^2 - 4}}{2}$$

$$\sqrt{x^2 + 5} = \sqrt{5} \sec z$$

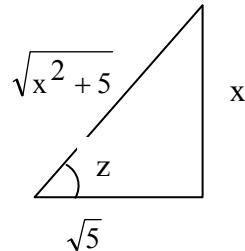
$$\sec z = \frac{\sqrt{x^2 + 5}}{\sqrt{5}}$$

Propiedad de los logaritmos

$$\int \sqrt{x^2 + 5} \, dx = \frac{1}{2} \left(x \sqrt{x^2 + 5} \right) + \frac{5}{2} \ln \left| \sqrt{x^2 + 5} + x \right| - \frac{5}{2} \ln \left| \sqrt{5} \right| + c$$

$$C_1 = -\frac{5}{2} \ln \left| \sqrt{5} \right| + c$$

$$\int \sqrt{x^2 + 5} \, dx = \frac{1}{2} \left(x \sqrt{x^2 + 5} \right) + \frac{5}{2} \ln \left| \sqrt{x^2 + 5} + x \right| + C_1$$



Resolver

$$\int \frac{dx}{x^3 \sqrt{x^2 - 9}}$$

$$\int \frac{dx}{x^3 \sqrt{x^2 - 9}} = \int \frac{(3 \sec z \tan z dz)}{(27 \sec^3 z) * 3 \tan z}$$

$$\frac{1}{27} \int \frac{dz}{\sec^2 z} = \frac{1}{27} \int \cos z^2 dz$$

Reemplazando la Identidad trigonométrica

$$\cos^2 z = \frac{1}{2} (1 + \cos 2z)$$

$$\frac{1}{27} \int \cos^2 z dz = \frac{1}{27} \int \frac{1}{2} (1 + \cos 2z) dz$$

$$\frac{1}{54} \int (1 + \cos 2z) dz$$

$$\frac{1}{54} \int dz + \frac{1}{54} \int \cos 2z dz$$

$$\frac{1}{54} z + \frac{1}{54} \left(\frac{1}{2} \sin 2z \right) + c$$

$$\frac{1}{54} z + \left(\frac{1}{108} \sin 2z \right) + c$$

Reemplazando la Identidad trigonométrica

$$\sin 2z = 2 \sin z \cos z$$

$$\frac{1}{54} z + \left(\frac{1}{108} \sin 2z \right) + c = \frac{1}{54} z + \frac{1}{108} (2 \sin z \cos z) + c$$

Reemplazando

$$\frac{1}{54} \left[\operatorname{arcsec} \left(\frac{x}{3} \right) \right] + \frac{1}{54} \left(\frac{\sqrt{x^2 - 9}}{x} * \frac{3}{x} \right)$$

$$\frac{1}{54} \operatorname{arcsec} \frac{x}{3} + \frac{1}{54} \left(\frac{3 \sqrt{x^2 - 9}}{x^2} \right) + c$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 9} = \sqrt{x^2 - 3^2} \Rightarrow x = 3 \sec z$$

$$x = 3 \sec z$$

$$X^2 = 9 \sec^2 z$$

$$X^3 = 27 \sec^3 z$$

$$\text{si } x = 3 \sec z \Rightarrow dx = 3 \sec z \tan z dz$$

$$\sqrt{x^2 - 9} = \sqrt{9 \sec^2 z - 9}$$

$$\sqrt{x^2 - 9} = \sqrt{9(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 9} = \sqrt{9(\tan^2 z)}$$

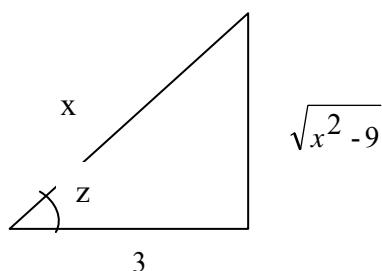
$$\sqrt{x^2 - 9} = 3 \tan z$$

$$\text{si } x = 3 \sec z \Rightarrow \sec z = \frac{x}{3}$$

$$z = \operatorname{arcsec} \left(\frac{x}{3} \right)$$

$$\text{si } \sec z = \frac{x}{3} \Rightarrow \cos z = \frac{3}{x}$$

$$\text{si } 3 \tan z = \sqrt{x^2 - 9} \Rightarrow \tan z = \frac{\sqrt{x^2 - 9}}{3}$$



$$\int \frac{dx}{x^3 \sqrt{x^2 - 9}} = \frac{1}{54} \operatorname{arcsec} \frac{x}{3} + \frac{1}{18} \left(\frac{\sqrt{x^2 - 9}}{x^2} \right) + c$$

Resolver

$$\int \frac{dx}{x^2 + 25}$$

$$\int \frac{dx}{x^2 + 25} = \int \frac{5 \sec^2 z dz}{25 \sec^2 z}$$

$$\int \frac{1}{5} dz$$

$$\frac{1}{5} \int dz = \frac{1}{5}(z) + c$$

Reemplazando

$$\int \frac{dx}{x^2 + 25} = \frac{1}{5}(z) + c$$

$$\int \frac{dx}{x^2 + 25} = \frac{1}{5} \operatorname{arc tg} \left(\frac{x}{5} \right) + c$$

$$x^2 + a^2 \Rightarrow x = a \operatorname{tg} z$$

$$x^2 + 25 = x^2 + 5^2 \Rightarrow x = 5 \operatorname{tg} z$$

$$x = 5 \operatorname{tg} z$$

$$x^2 = 25 \operatorname{tg}^2 z$$

$$\text{si } x = 5 \operatorname{tg} z \Rightarrow dx = 5 \sec^2 z dz$$

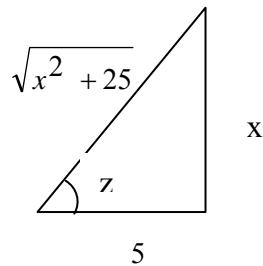
$$x^2 + 25 = 25 \operatorname{tg}^2 z + 25$$

$$x^2 + 25 = 25(\operatorname{tg}^2 z + 1)$$

$$x^2 + 25 = 25(\sec^2 z)$$

$$\text{si } x = 5 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{5}$$

$$z = \operatorname{arc tg} \left(\frac{x}{5} \right)$$



Resolver

$$\int \frac{x^2}{\sqrt{9-x^2}} dx$$

$$\int \frac{x^2}{\sqrt{9-x^2}} dx = \int \frac{9 \sin^2 z (3 \cos z dz)}{(3 \cos z)}$$

$$\int 9 \sin^2 z dz$$

$$\int 9 \sin^2 z dz = 9 \int \sin^2 z dz$$

Identidades trigonométricas

$$\frac{1 - \cos 2z}{2} = \sin^2 z$$

$$9 \int \sin^2 z dz = 9 \int \left(\frac{1 - \cos 2z}{2} \right) dz$$

$$\frac{9}{2} \int (1 - \cos 2z) dz = \left[\frac{9}{2} \left(\int 1 dz - \int \cos 2z dz \right) \right]$$

$$\frac{9}{2} \int dz - \frac{9}{2} \int \cos 2z dz$$

$$\frac{9}{2} z - \frac{9}{2} \left(\frac{1}{2} \sin 2z \right) + c$$

Identidades trigonométricas

$$\sin 2z = 2 \sin z * \cos z$$

$$\frac{9}{2} z - \frac{9}{2} \left(\frac{1}{2} \sin 2z \right) + c = \frac{9}{2} z - \frac{9}{4} (2 \sin z * \cos z) + c$$

Reemplazando

$$\frac{9}{2} z - \frac{9}{4} (2 \sin z * \cos z) + c = \frac{9}{2} z - \frac{9}{2} (\sin z * \cos z) + c$$

$$\frac{9}{2} z - \frac{9}{2} (\sin z * \cos z) + c = \frac{9}{2} \left(\arcsin \left(\frac{x}{3} \right) \right) - \frac{9}{2} \left[\frac{x}{3} * \left(\frac{\sqrt{9-x^2}}{3} \right) \right]$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{9-x^2} = \sqrt{3^2 - x^2}$$

$$\sqrt{3^2 - x^2} \Rightarrow x = 3 \sin z$$

$$x = 3 \sin z$$

$$x^2 = 9 \sin^2 z$$

$$\text{si } x = 3 \sin z \Rightarrow dx = 3 \cos z dz$$

$$\sqrt{9-x^2} = \sqrt{9 - 9 \sin^2 z}$$

$$\sqrt{9-x^2} = \sqrt{9 (1 - \sin^2 z)}$$

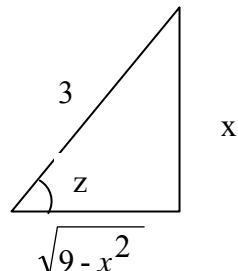
$$\sqrt{9-x^2} = \sqrt{9 (\cos^2 z)}$$

$$\sqrt{9-x^2} = 3 \cos z$$

$$\text{si } x = 3 \sin z \Rightarrow \sin z = \frac{x}{3}$$

$$z = \arcsin \left(\frac{x}{3} \right)$$

$$\text{si } \sqrt{9-x^2} = 3 \cos z \Rightarrow \cos z = \frac{\sqrt{9-x^2}}{3}$$



$$\frac{9}{2} \left(\arcsen \left(\frac{x}{3} \right) \right) - \frac{9}{2} \left[\frac{x}{3} * \left(\frac{\sqrt{9-x^2}}{3} \right) \right] + c = \frac{9}{2} \arcsen \left(\frac{x}{3} \right) - \frac{9}{2} \left(\frac{x \sqrt{9-x^2}}{9} \right) + c$$

$$\frac{9}{2} \arcsen \left(\frac{x}{3} \right) - \frac{9}{2} \left(\frac{x \sqrt{9-x^2}}{9} \right) + c = \frac{9}{2} \arcsen \left(\frac{x}{3} \right) - \frac{1}{2} \left(x \sqrt{9-x^2} \right) + c$$

$$\int \frac{x^2}{\sqrt{9-x^2}} dx = \frac{9}{2} \arcsen \left(\frac{x}{3} \right) - \frac{1}{2} \left(x \sqrt{9-x^2} \right) + c$$

Resolver

$$\int \sqrt{1-t^2} dt$$

$$\int \sqrt{1-t^2} dt = \int \cos z * (\cos z dz)$$

$$\int \cos^2 z dz$$

Identidades trigonométricas

$$\frac{1 + \cos 2z}{2} = \cos^2 z$$

$$\int \cos^2 z dz = \int \left(\frac{1 + \cos 2z}{2} \right) dz$$

$$\frac{1}{2} \int (1 + \cos 2z) dz = \frac{1}{2} \int 1 dz + \frac{1}{2} \int \cos 2z dz$$

$$\frac{1}{2}z + \frac{1}{2} \left(\frac{1}{2} \sin 2z \right) + c$$

$$\frac{1}{2}z + \frac{1}{4}(\sin 2z) + c$$

Identidades trigonométricas

$$\sin 2z = 2 \sin z * \cos z$$

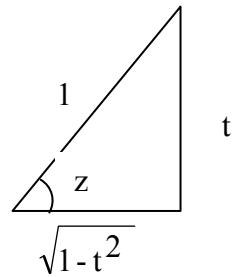
$$\frac{1}{2}z + \frac{1}{4}(\sin 2z) + c = \frac{1}{2}z + \frac{1}{4}(2 \sin z * \cos z) + c$$

Reemplazando

$$\frac{1}{2}z + \frac{1}{4}(2 \sin z * \cos z) + c = \frac{1}{2}z + \frac{1}{2}(\sin z * \cos z) + c$$

$$\frac{1}{2}z + \frac{1}{2}(\sin z * \cos z) + c = \frac{1}{2}(\arcsen(t)) + \frac{1}{2} \left[t * \sqrt{1-t^2} \right] + c$$

$$\begin{aligned} \sqrt{a^2 - t^2} &\Rightarrow t = a \sin z \\ \sqrt{1-t^2} &\Rightarrow t = \sin z \\ t &= \sin z \\ t^2 &= \sin^2 z \\ \text{si } t = \sin z &\Rightarrow dt = \cos z dz \\ \sqrt{1-t^2} &= \sqrt{1 - \sin^2 z} \\ \sqrt{1-t^2} &= \sqrt{(\cos^2 z)} \\ \sqrt{1-t^2} &= \cos z \\ t = \sin z &\Rightarrow z = \arcsen t \end{aligned}$$



Resolver

$$\int \frac{dx}{x^2 \sqrt{4+x^2}}$$

$$\int \frac{dx}{x^2 \sqrt{4+x^2}} = \int \frac{2 \sec^2 z dz}{(4 \tan^2 z)(2 \sec z)}$$

$$\frac{1}{4} \int \frac{\sec z}{\tan^2 z} dz$$

$$\frac{1}{4} \int \frac{1}{\frac{\sin z}{\cos^2 z}} dz$$

$$\frac{1}{4} \int \frac{\cos z}{\sin^2 z} dz$$

$$u = \sin z \quad du = \cos z \, dz$$

$$u^2 = \sin^2 z$$

Reemplazando

$$\frac{1}{4} \int \frac{\cos z \, dz}{\sin^2 z} = \frac{1}{4} \int \frac{du}{u^2}$$

$$\frac{1}{4} \int \frac{du}{u^2} = \frac{1}{4} \int u^{-2} \, du$$

$$\frac{1}{4(-1)} u^{-1} + c = -\frac{1}{4u} + c$$

Reemplazando

$$-\frac{1}{4u} + c = -\frac{1}{4(\sin z)} + c$$

$$-\frac{1}{4(\sin z)} + c = -\frac{1}{4 \left(\frac{x}{\sqrt{4+x^2}} \right)}$$

$$\int \frac{dx}{x^2 \sqrt{4+x^2}} = -\frac{1}{4 \left(\frac{x}{\sqrt{4+x^2}} \right)} + c = -\frac{\sqrt{4+x^2}}{4x} + c$$

$$\sqrt{a^2 + x^2} \Rightarrow x = a \tan z$$

$$\sqrt{4+x^2} = \sqrt{2^2 + x^2} \Rightarrow x = 2 \tan z$$

$$x = 2 \tan z$$

$$x^2 = 4 \tan^2 z$$

$$\text{si } x = 2 \tan z \Rightarrow dx = 2 \sec^2 z \, dz$$

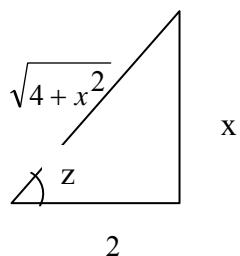
$$\sqrt{4+x^2} = \sqrt{4(1 + \tan^2 z)}$$

$$\sqrt{4+x^2} = \sqrt{4(\sec^2 z)}$$

$$\sqrt{4+x^2} = 2 \sec z$$

$$\text{si } x = 2 \tan z \Rightarrow \tan z = \frac{x}{2}$$

$$\sin z = \frac{x}{\sqrt{4+x^2}}$$



Resolver

$$\int \frac{dx}{3x^2 + 2x + 4}$$

Se debe completar el cuadrado

$$\int \frac{dx}{3x^2 + 2x + 4} = \int \frac{dx}{3\left(\frac{3x^2 + 2x + 4}{3}\right)}$$

$$\frac{1}{3} \int \frac{dx}{\left(\frac{3x^2}{3} + 2\frac{1}{3}x + \frac{4}{3}\right)}$$

$$\frac{1}{3} \int \frac{dx}{\left(x^2 + \frac{1}{3}2x + \frac{4}{3}\right)}$$

$$\frac{1}{3} \int \frac{dx}{\left[x^2 + \frac{1}{3}2x + \frac{1}{9} - \frac{1}{9} + \frac{4}{3}\right]}$$

$$\frac{1}{3} \int \frac{dx}{\left[\left(x^2 + \frac{1}{3}2x + \frac{1}{9}\right) - \frac{1}{9} + \frac{4}{3}\right]}$$

$$\frac{1}{3} \int \frac{dx}{\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{12}{9}\right]}$$

$$\frac{1}{3} \int \frac{dx}{\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right]}$$

Reemplazando

$$\frac{1}{3} \int \frac{dx}{\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right]} = \frac{1}{3} \int \frac{\sqrt{\frac{11}{9}} \sec^2 \theta d\theta}{\frac{11}{9} \sec^2 \theta} = \frac{1}{3} \frac{\sqrt{\frac{11}{9}}}{\frac{11}{9}} \int d\theta = \frac{1}{3} * \frac{\sqrt{\frac{11}{9}}}{\frac{11}{9}} \int d\theta = \frac{1}{3} * \frac{9\sqrt{11}}{3*11} \int d\theta = \frac{\sqrt{11}}{11} \int d\theta = \frac{\sqrt{11}}{11} \theta + c$$

$$\frac{1}{3} \int \frac{dx}{\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right]} = \frac{\sqrt{11}}{11} \theta + c = \frac{\sqrt{11}}{11} \operatorname{arc tg} \frac{3x+1}{\sqrt{11}} + c$$

$$\left(x^2 + a^2 \right) \Rightarrow x = a \operatorname{tg} \theta$$

$$\left(\left(x + \frac{1}{3}\right)^2 + \frac{11}{9} \right) = \left[\left(x + \frac{1}{3}\right)^2 + \left(\sqrt{\frac{11}{9}}\right)^2 \right]$$

$$\Rightarrow \left(x + \frac{1}{3}\right) = \sqrt{\frac{11}{9}} \operatorname{tg} \theta$$

$$\Rightarrow \left(x + \frac{1}{3}\right)^2 = \frac{11}{9} \operatorname{tg}^2 \theta$$

$$\text{si } \left(x + \frac{1}{3}\right) = \sqrt{\frac{11}{9}} \operatorname{tg} \theta \Rightarrow dx = \sqrt{\frac{11}{9}} \sec^2 \theta d\theta$$

$$\left(\left(x + \frac{1}{3}\right)^2 + \frac{11}{9} \right) = \left[\frac{11}{9} \operatorname{tg}^2 \theta + \frac{11}{9} \right]$$

$$\left(\left(x + \frac{1}{3}\right)^2 + \frac{11}{9} \right) = \left[\frac{11}{9} (\operatorname{tg}^2 \theta + 1) \right]$$

$$\left(\left(x + \frac{1}{3}\right)^2 + \frac{11}{9} \right) = \left[\frac{11}{9} (\sec^2 \theta) \right]$$

$$\Rightarrow \left(x + \frac{1}{3}\right) = \sqrt{\frac{11}{9}} \operatorname{tg} \theta \Rightarrow \operatorname{tg} \theta = \frac{x + \frac{1}{3}}{\sqrt{\frac{11}{9}}}$$

$$\Rightarrow \operatorname{tg} \theta = \frac{x + \frac{1}{3}}{\sqrt{\frac{11}{9}}} = \frac{\left(x + \frac{1}{3}\right)}{\frac{\sqrt{11}}{\sqrt{9}}} = \frac{\left(x + \frac{1}{3}\right)}{\frac{\sqrt{11}}{3}} = \frac{3\left(x + \frac{1}{3}\right)}{\sqrt{11}}$$

$$\text{si } \operatorname{tg} \theta = \frac{3x+1}{\sqrt{11}} \Rightarrow \theta = \operatorname{arc tg} \frac{3x+1}{\sqrt{11}}$$

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$$\int \frac{dx}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cos z dz}{\cos z}$$

$$\int dz$$

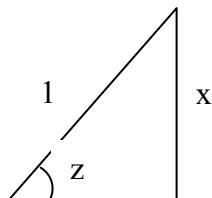
Tabla de integrales

$$\int dz = z + c$$

$$\int dz = z + c$$

Reemplazando

$$z + c = \arcsen(x) + c$$



$$\sqrt{1-x^2}$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sen z$$

$$\sqrt{1-x^2} \Rightarrow x = 1 \sen z$$

$$x = \sen z$$

$$x^2 = \sen^2 z$$

Si $x = \sen z \rightarrow dx = \cos z dz$

$$\sqrt{1-x^2} = \sqrt{1-\sen^2 z}$$

$$\sqrt{1-x^2} = \sqrt{(\cos^2 z)}$$

$$\sqrt{1-x^2} = \cos z$$

$$\text{si } x = \sen z$$

$$z = \arcsen(x)$$

$$\operatorname{tg} z = \frac{x}{\sqrt{1-x^2}}$$

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$$\int \frac{dx}{1+x^2} =$$

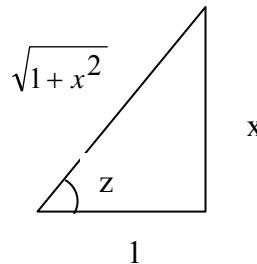
$$\int \frac{dx}{1+x^2} = \int \frac{\sec^2 z dz}{\sec^2 z} = \int dz$$

$$\int dz = (z) + c$$

Reemplazando

$$\int \frac{dx}{1+x^2} = (z) + c$$

$$\boxed{\int \frac{dx}{1+x^2} = \arctan x + c}$$



$$a^2 + x^2 \Rightarrow x = a \tan z$$

$$1+x^2 \Rightarrow x = \sqrt{1+\tan^2 z}$$

$$\begin{aligned} x &= \tan z \\ x^2 &= \tan^2 z \end{aligned}$$

$$\text{si } x = a \tan z \Rightarrow dx = \sec^2 z dz$$

$$1+x^2 = 1 + \tan^2 z$$

$$1+x^2 = (\sec^2 z)$$

$$\begin{aligned} \text{si } x &= \tan z \\ z &= \arctan x \end{aligned}$$

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$$\int \frac{dx}{x \sqrt{x^2 - 1}}$$

$$\int \frac{dx}{x \sqrt{x^2 - 1}} = \int \frac{\sec z \tan z dz}{(\sec z) \tan z}$$

$$\int dz = z + c$$

Tabla de integrales

$$\int dz = z + c$$

$$\int dz = z + c$$

Reemplazando

$$\int dz = z + c = \arcsin x + c$$

$$\int \frac{dx}{x \sqrt{x^2 - 1}} = \arcsin x + c$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 1} \Rightarrow x = 1 \sec z$$

$$x^2 = \sec^2 z$$

$$\text{Si } x = \sec z \rightarrow dx = \sec z \tan z dz$$

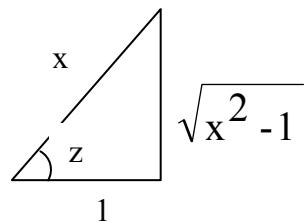
$$\sqrt{x^2 - 1} = \sqrt{\sec^2 z - 1}$$

$$\sqrt{x^2 - 1} = \sqrt{\tan^2 z}$$

$$\sqrt{x^2 - 1} = \tan z$$

$$\text{si } x = \sec z \Rightarrow \cos = \frac{1}{x}$$

$$z = \arcsin x$$



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$$\int \frac{dx}{\sqrt{4-x^2}}$$

$$\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2 \cos z \ dz}{(2 \cos z)}$$

$$\int dz = z + c$$

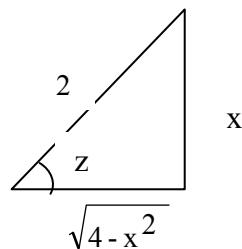
Tabla de integrales

$$\int dz = z + c$$

Reemplazando

$$\int dz = z + c = \arcsen \frac{x}{2} + c$$

$$\int \frac{dx}{\sqrt{4-x^2}} = \arcsen \frac{x}{2} + c$$



$$\sqrt{a^2 - x^2} \Rightarrow x = a \sen z$$

$$\sqrt{4 - x^2} \Rightarrow x = 2 \sen z$$

$$x = 2 \sen z$$

$$x^2 = 4 \sen^2 z$$

$$\text{si } x = 2 \sen z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4 - x^2} = \sqrt{4 - 4 \sen^2 z}$$

$$\sqrt{4 - x^2} = \sqrt{4(1 - \sen^2 z)}$$

$$\sqrt{4 - x^2} = \sqrt{4(\cos^2 z)}$$

$$\sqrt{4 - x^2} = 2 \cos z$$

$$\text{si } x = 2 \sen z \Rightarrow \sen z = \frac{x}{2}$$

$$z = \arcsen \frac{x}{2}$$

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$$\int \frac{dx}{9+x^2} =$$

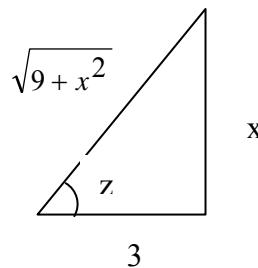
$$\int \frac{dx}{9+x^2} = \int \frac{3 \sec^2 z dz}{9 \sec^2 z} = \frac{1}{3} \int dz$$

$$\frac{1}{3} \int dz = \frac{1}{3}(z) + c$$

Reemplazando

$$\int \frac{dx}{1+x^2} = \frac{1}{3}(z) + c$$

$$\boxed{\int \frac{dx}{9+x^2} = \frac{1}{3} \operatorname{arc \, tg} \frac{x}{3} + c}$$



$$a^2 + x^2 \Rightarrow x = a \operatorname{tg} z$$

$$9 + x^2 = 3^2 + x^2 \Rightarrow x = 3 \operatorname{tg} z$$

$$\begin{aligned} x &= 3 \operatorname{tg} z \\ x^2 &= 9 \operatorname{tg}^2 z \end{aligned}$$

$$\text{si } x = 3 \operatorname{tg} z \Rightarrow dx = 3 \sec^2 z dz$$

$$9 + x^2 = 9 + 9 \operatorname{tg}^2 z$$

$$\begin{aligned} 9 + x^2 &= 9(1 + \operatorname{tg}^2 z) \\ 9 + x^2 &= 9 \left(\sec^2 z \right) \end{aligned}$$

$$\text{si } x = 3 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{3}$$

$$z = \operatorname{arc \, tg} \frac{x}{3}$$

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$$\int \frac{dx}{\sqrt{25 - 16x^2}}$$

$$\int \frac{dx}{\sqrt{25 - 16x^2}} = \int \frac{\frac{5}{4} \cos z \ dz}{(5 \cos z)}$$

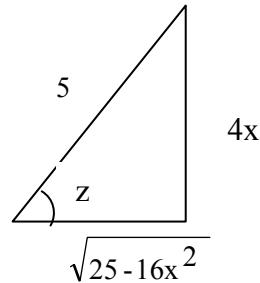
$$\frac{1}{4} \int dz$$

Tabla de integrales

$$\int dz = z + c$$

$$\frac{1}{4} \int dz = z + c = \arcsen \frac{4x}{5} + c$$

$$\int \frac{dx}{\sqrt{25 - 16x^2}} = \frac{1}{4} \arcsen \frac{4x}{5} + c$$



$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{25 - 16x^2} = \sqrt{5^2 - (4x)^2} \Rightarrow 4x = 5 \sin z$$

$$4x = 5 \sin z$$

$$(4x)^2 = 25 \sin^2 z$$

$$\text{si } 4x = 5 \sin z \Rightarrow 4 dx = 5 \cos z dz$$

$$dx = \frac{5}{4} \cos z dz$$

$$\sqrt{25 - (4x)^2} = \sqrt{25 - 25 \sin^2 z}$$

$$\sqrt{25 - (4x)^2} = \sqrt{25 (1 - \sin^2 z)}$$

$$\sqrt{25 - (4x)^2} = \sqrt{25 (\cos^2 z)}$$

$$\sqrt{25 - 16x^2} = 5 \cos z$$

$$\text{si } 4x = 5 \sin z \Rightarrow \sin z = \frac{4x}{5}$$

$$z = \arcsen \frac{4x}{5}$$

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$$\int \frac{dx}{4x^2 + 9} =$$

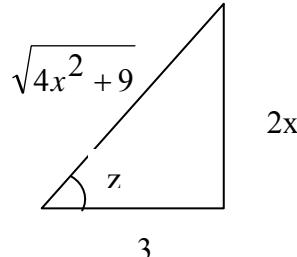
$$\int \frac{dx}{4x^2 + 9} = \int \frac{\frac{3}{2} \sec^2 z dz}{9 \sec^2 z} = \frac{3}{18} \int dz$$

$$\frac{1}{6} \int dz = \frac{1}{6}(z) + c$$

Reemplazando

$$\int \frac{dx}{4x^2 + 9} = \frac{1}{6}(z) + c$$

$$\int \frac{dx}{4x^2 + 9} = \frac{1}{6} \operatorname{arc \, tg} \frac{2x}{3} + c$$



$$a^2 + x^2 \Rightarrow x = a \operatorname{tg} z$$

$$4x^2 + 9 = (2x)^2 + 3^2 \Rightarrow 2x = 3 \operatorname{tg} z$$

$$\begin{aligned} 2x &= 3 \operatorname{tg} z \\ (2x)^2 &= 9 \operatorname{tg}^2 z \end{aligned}$$

$$\text{si } 2x = 3 \operatorname{tg} z \Rightarrow 2 dx = 3 \sec^2 z dz$$

$$dx = \frac{3}{2} \sec^2 z dz$$

$$(2x)^2 + 9 = 9 \operatorname{tg}^2 z + 9$$

$$(2x)^2 + 9 = 9(\operatorname{tg}^2 z + 1)$$

$$(2x)^2 + 9 = 9(\sec^2 z)$$

$$\text{si } 2x = 3 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{2x}{3}$$

$$z = \operatorname{arc \, tg} \frac{2x}{3}$$

$$\int \frac{dx}{x \sqrt{4x^2 - 9}}$$

$$\int \frac{dx}{x \sqrt{4x^2 - 9}} = \int \frac{\frac{3}{2} \sec z \tan z dz}{\left(\frac{3}{2} \sec z\right)^* 3 \tan z}$$

$$\frac{1}{3} \int dz = \frac{1}{3} z + c$$

Tabla de integrales

$$\int dz = z + c$$

$$\frac{1}{3} \int dz = \frac{1}{3} z + c$$

Reemplazando

$$\frac{1}{3} \int dz = \frac{1}{3} z + c = \frac{1}{3} \operatorname{arc sec} \frac{2x}{3} + c$$

$$\int \frac{dx}{x \sqrt{4x^2 - 9}} = \frac{1}{3} \operatorname{arc sec} \frac{2x}{3} + c$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{4x^2 - 9} = \sqrt{(2x)^2 - 3^2} \Rightarrow 2x = 3 \sec z$$

$$(2x)^2 = 9 \sec^2 z$$

$$\text{Si } 2x = 3 \sec z \rightarrow 2 dx = 3 \sec z \tan z dz$$

$$x = \frac{3}{2} \sec z$$

$$dx = \frac{3}{2} \sec z \tan z dz$$

$$\sqrt{4x^2 - 9} = \sqrt{9 \sec^2 z - 9}$$

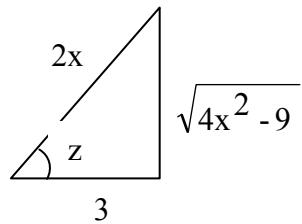
$$\sqrt{4x^2 - 9} = \sqrt{9(\sec^2 z - 1)}$$

$$\sqrt{4x^2 - 9} = \sqrt{9 \tan^2 z}$$

$$\sqrt{4x^2 - 9} = 3 \tan z$$

$$\text{si } x = \frac{3}{2} \sec z \Rightarrow \sec = \frac{2x}{3}$$

$$z = \operatorname{arc sec} \frac{2x}{3}$$



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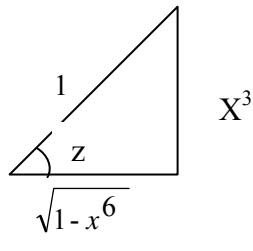
$$\int \frac{x^2 dx}{\sqrt{1-x^6}}$$

$$\int \frac{x^2 dx}{\sqrt{1-x^6}} = \int \frac{\frac{1}{3} \cos z dz}{(\cos z)}$$

$$\frac{1}{3} \int dz$$

Reemplazando

$$\frac{1}{3} \int dz = \frac{1}{3} z + c = \frac{1}{3} \arcsen x^3 + c$$



$$\sqrt{a^2 - x^2} \Rightarrow x = a \sen z$$

$$\sqrt{1 - x^6} = \sqrt{1^2 - (x^3)^2} \Rightarrow x^3 = 1 \sen z$$

$$x^3 = \sen z$$

$$\text{si } x^3 = \sen z \Rightarrow 3x^2 dx = \cos z dz$$

$$x^2 dx = \frac{1}{3} \cos z dz$$

$$\sqrt{1 - (x^3)^2} = \sqrt{1 - (\sen z)^2}$$

$$\sqrt{1 - (x^3)^2} = \sqrt{1 - \sen^2 z}$$

$$\sqrt{1 - (x^3)^2} = \sqrt{\cos^2 z}$$

$$\sqrt{1 - (x^3)^2} = \cos z$$

$$\text{si } x^3 = \sen z$$

$$z = \arcsen x^3$$

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$$\int \frac{x \, dx}{x^4 + 3} =$$

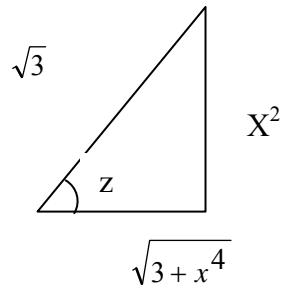
$$\int \frac{x \, dx}{x^4 + 3} = \int \frac{\frac{\sqrt{3}}{2} \sec^2 z \, dz}{3 \sec^2 z} = \frac{\sqrt{3}}{6} \int dz$$

$$\frac{\sqrt{3}}{6} \int dz = \frac{\sqrt{3}}{6} (z) + c$$

Reemplazando

$$\int \frac{x \, dx}{x^4 + 3} = \frac{\sqrt{3}}{6} (z) + c$$

$$\int \frac{x \, dx}{x^4 + 3} = \frac{\sqrt{3}}{6} \arctan \left(\frac{\sqrt{3} x^2}{3} \right) + c$$



$$a^2 + x^2 \Rightarrow x = a \tan z$$

$$x^4 + 3 = (x^2)^2 + (\sqrt{3})^2 \Rightarrow x^2 = \sqrt{3} \tan z$$

$$x^2 = \sqrt{3} \tan z$$

$$\text{si } x^2 = \sqrt{3} \tan z \Rightarrow 2x \, dx = \sqrt{3} \sec^2 z \, dz$$

$$x \, dx = \frac{\sqrt{3}}{2} \sec^2 z \, dz$$

$$x^4 + 3 = (x^2)^2 + (\sqrt{3})^2 = [\sqrt{3} \tan z]^2 + 3$$

$$x^4 + 3 = 3 \tan^2 z + 3$$

$$x^4 + 3 = 3(\tan^2 z + 1)$$

$$x^4 + 3 = 3 \sec^2 z$$

$$\text{si } x^2 = \sqrt{3} \tan z \Rightarrow \tan z = \frac{x^2}{\sqrt{3}}$$

$$z = \arctan \frac{x^2}{\sqrt{3}}$$

$$z = \arctan \frac{x^2 * \sqrt{3}}{\sqrt{3} * \sqrt{3}} =$$

$$z = \arctan \frac{\sqrt{3} x^2}{3} =$$

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$$\int \frac{dx}{x \sqrt{x^4 - 1}} =$$

Se multiplica por $2x$

$$\int \frac{dx}{x \sqrt{x^4 - 1}} = \int \frac{(2x)dx}{(2x)x \sqrt{x^4 - 1}} = \int \frac{2x dx}{2x^2 \sqrt{x^4 - 1}}$$

$$\frac{1}{2} \int \frac{2x dx}{x^2 \sqrt{x^4 - 1}} = \frac{1}{2} \int \frac{\sec z \tan z dz}{(\sec z) \tan z}$$

$$\frac{1}{2} \int dz = \frac{1}{2} z + c$$

Tabla de integrales

$$\int dz = z + c$$

Reemplazando

$$\frac{1}{2} \int dz = \frac{1}{2} z + c = \frac{1}{2} \operatorname{arcsec} x^2 + c$$

$$\int \frac{dx}{x \sqrt{x^4 - 1}} = \frac{1}{2} \operatorname{arcsec} x^2 + c$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^4 - 1} \quad \sqrt{(\sec z)^2 - 1} \Rightarrow x^2 = 1 \sec z$$

$$x^2 = \sec z$$

$$\text{Si } x^2 = \sec z \rightarrow 2x dx = \sec z \tan z dz$$

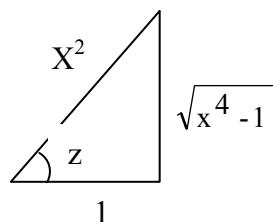
$$\sqrt{(\sec z)^2 - 1} = \sqrt{(\sec z)^2 - 1}$$

$$\sqrt{(\sec z)^2 - 1} = \sqrt{\tan^2 z}$$

$$\sqrt{x^4 - 1} = \tan z$$

$$\text{si } x^2 = \sec z \Rightarrow \cos = \frac{1}{x^2}$$

$$z = \operatorname{arcsec} x^2$$



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$$\int \frac{dx}{\sqrt{4-(x+2)^2}}$$

$$\int \frac{dx}{\sqrt{4-(x+2)^2}} = \int \frac{2 \cos z \ dz}{(2 \cos z)}$$

$$\int dz = z + c$$

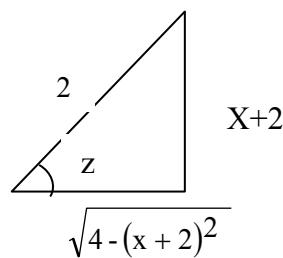
Tabla de integrales

$$\int dz = z + c$$

Reemplazando

$$\int dz = z + c = \arcsen \frac{x+2}{2} + c$$

$$\int \frac{dx}{\sqrt{4-(x+2)^2}} = \arcsen \frac{x+2}{2} + c$$



$$\sqrt{a^2 - x^2} \Rightarrow x = a \sen z$$

$$\sqrt{4-(x+2)^2} = \sqrt{2^2 - (x+2)^2} \Rightarrow x+2 = 2 \sen z$$

$$x+2 = 2 \sen z$$

$$(x+2)^2 = 4 \sen^2 z$$

$$\text{si } x+2 = 2 \sen z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4-(x+2)^2} = \sqrt{4 - 4 \sen^2 z}$$

$$\sqrt{4-(x+2)^2} = \sqrt{4(1 - \sen^2 z)}$$

$$\sqrt{4-(x+2)^2} = \sqrt{4(\cos^2 z)}$$

$$\sqrt{4-(x+2)^2} = 2 \cos z$$

$$\text{si } x+2 = 2 \sen z \Rightarrow \sen z = \frac{x+2}{2}$$

$$z = \arcsen \frac{x+2}{2}$$

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$$\int \frac{dx}{e^x + e^{-x}} =$$

Se multiplica por e^x

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{(e^x)dx}{(e^x)*(e^x + e^{-x})} = \int \frac{e^x dx}{e^{2x} + e^0} = \int \frac{e^x dx}{e^{2x} + 1}$$

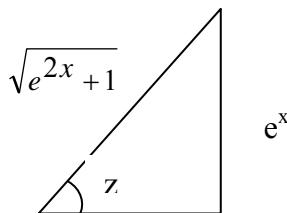
$$\int \frac{e^x dx}{(e^x)^2 + 1} = \int \frac{\sec^2 z dz}{\sec^2 z} = \int dz$$

$$\int dz = (z) + c$$

Reemplazando

$$\int \frac{dx}{e^x + e^{-x}} = z + c$$

$$\int \frac{dx}{e^x + e^{-x}} = \arctan e^x + c$$



1

$$a^2 + x^2 \Rightarrow x = a \tan z$$

$$(e^x)^2 + 1 \Rightarrow e^x = 1 \tan z$$

$$e^x = \tan z$$

$$(e^x)^2 = \tan^2 z$$

$$\text{si } e^x = \tan z \Rightarrow e^x dx = \sec^2 z dz$$

$$(e^x)^2 + 1 = (\tan z)^2 + 1$$

$$(e^x)^2 + 1 = \tan^2 z + 1$$

$$(e^x)^2 + 1 = \sec^2 z$$

$$\begin{aligned} \text{si } & e^x = \tan z \\ & z = \arctan e^x \end{aligned}$$

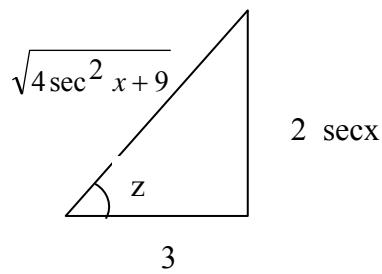
62 Pág. 246 Calculo diferencial e integral SCHAUM

$$\int \frac{\sec x \tan x \, dx}{9 + 4\sec^2 x} =$$

$$\int \frac{dx}{9 + 4 \sec^2 x} = \int \frac{\frac{3}{2} \sec^2 z \, dz}{9 \sec^2 z} = \frac{3}{18} \int dz$$

$$\frac{1}{6} \int dz = \frac{1}{6}(z) + c$$

$$\frac{1}{6}(z) + c = \frac{1}{6} \arctan \left(\frac{2 \sec x}{3} \right) + c$$



$$a^2 + x^2 \Rightarrow x = a \tan z$$

$$9 + 4 \sec^2 x = (3)^2 + (2 \sec x)^2 \Rightarrow 2 \sec x = 3 \tan z$$

$$\begin{aligned} 2 \sec x &= 3 \tan z \\ (2 \sec x)^2 &= 9 \tan^2 z \end{aligned}$$

$$\text{si } 2 \sec x = 3 \tan z \Rightarrow 2 \sec x \tan x \, dx = 3 \sec^2 z \, dz$$

$$\sec x \tan x \, dx = \frac{3}{2} \sec^2 z \, dz$$

$$9 + (2 \sec x)^2 = 9 + 9 \tan^2 z$$

$$9 + (2 \sec x)^2 = 9(1 + \tan^2 z)$$

$$9 + (2 \sec x)^2 = 9(\sec^2 z)$$

$$\text{si } 2 \sec x = 3 \tan z \Rightarrow \tan z = \frac{2 \sec x}{3}$$

$$z = \arctan \frac{2 \sec x}{3}$$

$$\int \frac{(x+3)dx}{\sqrt{1-x^2}}$$

$$\int \frac{(x+3)dx}{\sqrt{1-x^2}} = \int \frac{(x+3)\cos z dz}{(\cos z)} = \int (x+3)dz$$

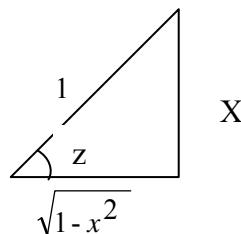
$$\int (\sin z + 3)dz = \int \sin z dz + \int 3 dz = -\cos z + 3z + C$$

Ordenando

$$3z - \cos z + C$$

Reemplazando

$$\int \frac{(x+3)dz}{\sqrt{1-x^2}} dz = 3z - \cos z + C = 3 \arcsen x^3 - \sqrt{1-x^2} + C$$



$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{1-x^2} \Rightarrow x = 1 \sin z$$

$$\begin{aligned} x &= \sin z \\ x^2 &= \sin^2 z \end{aligned}$$

$$\text{si } x = \sin z \Rightarrow dx = \cos z dz$$

$$\sqrt{1-x^2} = \sqrt{1 - (\sin z)^2}$$

$$\sqrt{1-x^2} = \sqrt{1 - \sin^2 z}$$

$$\sqrt{1-x^2} = \sqrt{\cos^2 z}$$

$$\sqrt{1-x^2} = \cos z$$

$$\text{si } x = \sin z$$

$$z = \arcsen x^3$$

$$\cos z = \sqrt{1-x^2}$$

$$\int \frac{(2x - 7) dx}{x^2 + 9} =$$

$$\int \frac{(2x - 7) dx}{x^2 + 9} = \int \frac{(2x - 7) * 3 \sec^2 z dz}{9 \sec^2 z} = \frac{3}{9} \int (2x - 7) dz$$

$$\frac{1}{3} \int (2x - 7) dz = \frac{1}{3} \int 2x dz - \frac{1}{3} \int 7 dz$$

Reemplazando el valor de x

$$\frac{1}{3} \int 2x dz - \frac{1}{3} \int 7 dz = \frac{2}{3} \int 3 \operatorname{tg} z dz - \frac{7}{3} \int dz = 2 \int \operatorname{tg} z dz - \frac{7}{3} \int dz$$

$$2 \int \operatorname{tg} z dz - \frac{7}{3} \int dz =$$

Tabla de integrales

$$\int \operatorname{tg} z dz = \ln|\sec z| + c$$

$$\int dz = z + c$$

$$2 \int \operatorname{tg} z dz - \frac{7}{3} \int dz = 2 \ln|\sec z| - \frac{7}{3} z + c$$

Reemplazando

$$2 \ln|\sec z| - \frac{7}{3} z + c = 2 \ln \left| \frac{\sqrt{x^2 + 9}}{3} \right| - \frac{7}{3} \operatorname{arctg} \left(\frac{x}{3} \right) + c$$

$$\frac{2}{3} \ln \left| \frac{\left(\sqrt{x^2 + 9} \right)^2}{(3)^2} \right| - \frac{7}{3} \operatorname{arctg} \frac{x}{3} + c = \ln \left| \frac{x^2 + 9}{9} \right| - \frac{7}{3} \operatorname{arctg} \frac{x}{3} + c$$

$$\ln|x^2 + 9| - \ln|9| - \frac{7}{3} \operatorname{arctg} \frac{x}{3} + c$$

pero: $-\ln|9| + c = C_1$

Reemplazando

$$\ln|x^2 + 9| - \frac{7}{3} \operatorname{arctg} \frac{x}{3} + C_1$$

$$a^2 + x^2 \Rightarrow x = a \operatorname{tg} z$$

$$x^2 + 9 = (x)^2 + 3^2 \Rightarrow x = 3 \operatorname{tg} z$$

$$x = 3 \operatorname{tg} z$$

$$(x)^2 = 9 \operatorname{tg}^2 z$$

$$\text{si } x = 3 \operatorname{tg} z \Rightarrow dx = 3 \sec^2 z dz$$

$$(x)^2 + 9 = 9 \operatorname{tg}^2 z + 9$$

$$(x)^2 + 9 = 9(\operatorname{tg}^2 z + 1)$$

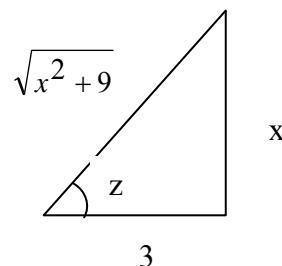
$$(x)^2 + 9 = 9(\sec^2 z)$$

$$\sec^2 z = \frac{x^2 + 9}{9} \Rightarrow \sec z = \frac{\sqrt{x^2 + 9}}{\sqrt{9}}$$

$$\sec z = \frac{\sqrt{x^2 + 9}}{3}$$

$$\text{si } x = 3 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x}{3}$$

$$z = \operatorname{arc tg} \frac{x}{3}$$



$$\int \frac{dy}{y^2 + 10y + 30} =$$

Se completa el cuadrado, se adiciona 25 y se le resta 25

$$\int \frac{dy}{y^2 + 10y + 30} = \int \frac{dy}{y^2 + 2(5)y + 25 + 30 - 25} = \int \frac{dy}{(y^2 + 10y + 25) + 30 - 25}$$

$$\int \frac{dy}{(y^2 + 10y + 25) + 5} = \int \frac{dy}{(y + 5)^2 + 5}$$

$$\int \frac{dy}{(y + 5)^2 + 5} = \int \frac{\sqrt{5} \sec^2 z dz}{5 \sec^2 z} = \frac{\sqrt{5}}{5} \int dz$$

Tabla de integrales

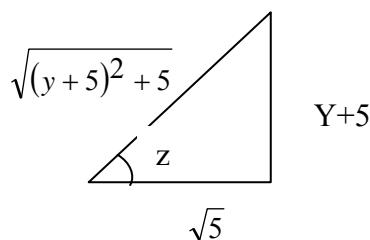
$$\int dz = z + c$$

$$\frac{\sqrt{5}}{5} \int dz = \frac{\sqrt{5}}{5} z + c$$

Reemplazando

$$\frac{\sqrt{5}}{5} z + c = \frac{\sqrt{5}}{5} \operatorname{arc tg} \left(\frac{\sqrt{5} y + 5\sqrt{5}}{5} \right) + c$$

$$\int \frac{dy}{y^2 + 10y + 30} = \frac{\sqrt{5}}{5} \operatorname{arc tg} \left(\frac{\sqrt{5} y + 5\sqrt{5}}{5} \right) + c$$



$$y^2 + a^2 \Rightarrow y = a \operatorname{tg} z$$

$$(y + 5)^2 + 5 = (y + 5)^2 + (\sqrt{5})^2 \Rightarrow y + 5 = \sqrt{5} \operatorname{tg} z$$

$$y + 5 = \sqrt{5} \operatorname{tg} z$$

$$(y + 5)^2 = 5 \operatorname{tg}^2 z$$

$$\text{si } y + 5 = \sqrt{5} \operatorname{tg} z \Rightarrow dy = \sqrt{5} \sec^2 z dz$$

$$(y + 5)^2 + 5 = 5 \operatorname{tg}^2 z + 5$$

$$(y + 5)^2 + 5 = 5(\operatorname{tg}^2 z + 1)$$

$$(y + 5)^2 + 5 = 5(\sec^2 z)$$

$$\text{si } y + 5 = \sqrt{5} \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{y + 5}{\sqrt{5}}$$

$$z = \operatorname{arc tg} \frac{y + 5}{\sqrt{5}} = \operatorname{arc tg} \frac{\sqrt{5} (y + 5)}{\sqrt{5} * \sqrt{5}}$$

$$z = \operatorname{arc tg} \frac{\sqrt{5} (y + 5)}{5} = \operatorname{arc tg} \frac{\sqrt{5} y + 5\sqrt{5}}{5}$$

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$$\int \frac{dx}{\sqrt{20+8x-x^2}}$$

Se ordena

$$\int \frac{dx}{\sqrt{20+8x-x^2}} = \int \frac{dx}{\sqrt{-\left(x^2 - 8x - 20\right)}} = \int \frac{dx}{\sqrt{-\left(x^2 - 2(4)x - 20\right)}}$$

se completa el cuadrado, sumando 16 y restando 16

$$\int \frac{dx}{\sqrt{-\left(x^2 - 8x + 16 - 20 - 16\right)}} = \int \frac{dx}{\sqrt{-\left[\left(x^2 - 8x + 16\right) - 20 - 16\right]}}$$

$$\int \frac{dx}{\sqrt{-\left[\left(x^2 - 8x + 16\right) - 36\right]}} = \int \frac{dx}{\sqrt{-\left[\left(x - 4\right)^2 - 36\right]}}$$

Se ordena y luego se reemplaza

$$\int \frac{dx}{\sqrt{36 - (x - 4)^2}} = \int \frac{6 \cos z dz}{6 \cos z} = \int dz$$

Tabla de integrales

$$\int dz = z + c$$

se reemplaza

$$\int dz = z + c = \arcsen\left(\frac{x - 4}{6}\right) + c$$

$$\int \frac{dx}{\sqrt{20+8x-x^2}} = \arcsen\left(\frac{x - 4}{6}\right) + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{6^2 - (x - 4)^2} \Rightarrow x - 4 = 6 \sin z$$

$$x - 4 = 6 \sin z$$

$$(x - 4)^2 = 36 \sin^2 z$$

$$\text{si } x - 4 = 6 \sin z \Rightarrow dx = 6 \cos z dz$$

$$\sqrt{36 - (x - 4)^2} = \sqrt{36 - (36 \sin^2 z)}$$

$$\sqrt{36 - (x - 4)^2} = \sqrt{36(1 - \sin^2 z)}$$

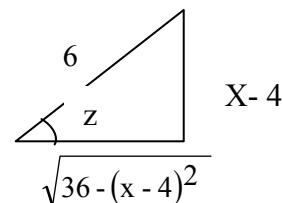
$$\sqrt{36 - (x - 4)^2} = \sqrt{36(\cos^2 z)}$$

$$\sqrt{36 - (x - 4)^2} = 6 \cos z$$

$$\text{si } x - 4 = 6 \sin z \Rightarrow \sin z = \frac{x - 4}{6}$$

$$z = \arcsen\left(\frac{x - 4}{6}\right)$$

$$\cos z = \sqrt{1 - \sin^2 z}$$



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$$\int \frac{dx}{2x^2 + 2x + 5} =$$

Se multiplica por 2 y se divide por 2 para intentar completar el cuadrado

$$\int \frac{dx}{2x^2 + 2x + 5} = \int \frac{(2)dx}{(2)[2x^2 + 2x + 5]} = \int \frac{2 dx}{[4x^2 + (2)2x + 10]} = \int \frac{2 dx}{[(2x)^2 + (2)(2)(1)(x) + 10]}$$

se completa el cuadrado, sumando 1 y restando 1

$$\int \frac{2 dx}{[(2x)^2 + 4x + 1 + 10 - 1]} = \int \frac{2 dx}{[(2x)^2 + 4x + 1] + 10 - 1} = \int \frac{2 dx}{[(2x)^2 + 4x + 1] + 9} = \int \frac{2 dx}{[(2x + 1)^2 + 9]}$$

$$\int \frac{2 dx}{(2x + 1)^2 + 3^2} =$$

reemplazando

$$\int \frac{2 dx}{(2x + 1)^2 + 3^2} = \int \frac{3 \sec^2 z dz}{9 \sec^2 z} = \frac{1}{3} \int dz$$

$$a^2 + x^2 \Rightarrow x = a \operatorname{tg} z$$

$$(2x + 1)^2 + 3^2 \Rightarrow 2x + 1 = 3 \operatorname{tg} z$$

$$2x + 1 = 3 \operatorname{tg} z \rightarrow 2x = 3 \operatorname{tg} z - 1$$

$$(2x+1)^2 = 9 \operatorname{tg}^2 z$$

$$\text{si } 2x + 1 = 3 \operatorname{tg} z \Rightarrow 2 dx = 3 \sec^2 z dz$$

$$(2x + 1)^2 + 9 = 9 \operatorname{tg}^2 z + 9$$

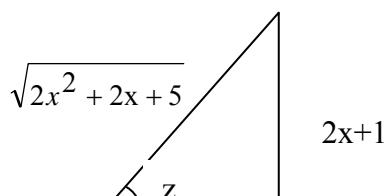
$$(2x + 1)^2 + 9 = 9(\operatorname{tg}^2 z + 1)$$

$$(2x + 1)^2 + 9 = 9(\sec^2 z)$$

$$\sec^2 z = \frac{(2x + 1)^2 + 10}{9}$$

$$\text{si } 2x + 1 = 3 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{2x + 1}{3}$$

$$z = \operatorname{arc} \operatorname{tg} \frac{2x + 1}{3}$$



3

$$\int \frac{(x+1) dx}{x^2 - 4x + 8} =$$

se completa el cuadrado, sumando 4 y restando 4

$$\int \frac{(x+1) dx}{x^2 - 4x + 8} = \int \frac{(x+1) dx}{x^2 - 4x + 4 + 8 - 4} = \int \frac{(x+1) dx}{(x^2 - 4x + 4) + 8 - 4}$$

$$\int \frac{(x+1) dx}{(x^2 - 4x + 4) + 4} = \int \frac{(x+1) dx}{(x-2)^2 + 4} = \int \frac{x dx}{(x-2)^2 + 4} + \int \frac{1 dx}{(x-2)^2 + 4}$$

$$\int \frac{x dx}{(x-2)^2 + 4} + \int \frac{1 dx}{(x-2)^2 + 4} =$$

$$\int \frac{(2 \operatorname{tg} z + 2)(2 \sec^2 z dz)}{4 \sec^2 z} + \int \frac{2 \sec^2 z dz}{4 \sec^2 z} =$$

$$\int \frac{(2 \operatorname{tg} z + 2)(2 dz)}{4} + \int \frac{2 dz}{4} =$$

$$\frac{1}{2} \int (2 \operatorname{tg} z + 2) dz \cdot \frac{1}{2} \int dz = \frac{1}{2} \int 2 \operatorname{tg} z dz + \frac{1}{2} \int 2 dz + \frac{1}{2} \int dz$$

$$\frac{2}{2} \int \operatorname{tg} z dz + \frac{2}{2} \int dz + \frac{1}{2} \int dz = \int \operatorname{tg} z dz + \int dz + \frac{1}{2} \int dz$$

$$\int \operatorname{tg} z dz + \frac{3}{2} \int dz =$$

Tabla de integrales

$$\int \operatorname{tg} z dz = \ln |\sec z| + c$$

$$\int dz = z + c$$

$$\int \operatorname{tg} z dz + \frac{3}{2} \int dz = \ln |\sec z| + \frac{3}{2} z + c$$

Reemplazando

$$\ln |\sec z| + \frac{3}{2} z + c = \ln \left| \frac{\sqrt{x^2 - 4x + 8}}{4} \right| + \frac{3}{2} \operatorname{arctg} \left(\frac{x-2}{2} \right) + c$$

$$a^2 + x^2 \Rightarrow x = a \operatorname{tg} z$$

$$(x-2)^2 + 2^2 \Rightarrow x-2 = 2 \operatorname{tg} z$$

$$\begin{aligned} x-2 &= 2 \operatorname{tg} z \rightarrow x = 2 \operatorname{tg} z + 2 \\ (x-2)^2 &+ 4 = 4 \operatorname{tg}^2 z + 4 \end{aligned}$$

$$\text{si } x-2 = 2 \operatorname{tg} z \Rightarrow dx = 2 \sec^2 z dz$$

$$(x-2)^2 + 4 = 4 \operatorname{tg}^2 z + 4$$

$$(x-2)^2 + 4 = 4(\operatorname{tg}^2 z + 1)$$

$$(x-2)^2 + 4 = 4(\sec^2 z)$$

$$\sec^2 z = \frac{(x-2)^2 + 4}{4} = \frac{x^2 - 4x + 8}{4}$$

$$\sec z = \frac{\sqrt{x^2 - 4x + 8}}{4}$$

$$\text{si } x-2 = 2 \operatorname{tg} z \Rightarrow \operatorname{tg} z = \frac{x-2}{2}$$

$$z = \operatorname{arc} \operatorname{tg} \frac{x-2}{2}$$

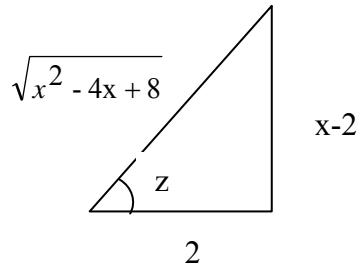
$$\frac{1}{2} \ln \left| \frac{\left(\sqrt{x^2 - 4x + 8} \right)^2}{(2)^2} \right| + \frac{3}{2} \arctg \frac{x-2}{2} + c = \frac{1}{2} \ln \left| \frac{x^2 - 4x + 8}{4} \right| + \frac{3}{2} \arctg \frac{x-2}{2} + c$$

$$\frac{1}{2} \ln |x^2 - 4x + 8| - \ln |4| + \frac{3}{2} \arctg \frac{x-2}{2} + c$$

pero: $-\ln |4| + c = C_1$

Reemplazando

$$\frac{1}{2} \ln |x^2 - 4x + 8| + \frac{3}{2} \arctg \frac{x-2}{2} + C_1$$



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$$\int \frac{dx}{\sqrt{28 - 12x - x^2}}$$

Se ordena

$$\int \frac{dx}{\sqrt{28 - 12x - x^2}} = \int \frac{dx}{\sqrt{- (x^2 + 12x - 28)}} = \int \frac{dx}{\sqrt{- (x^2 + 2(6)x - 28)}}$$

se completa el cuadrado, sumando 36 y restando 36

$$\int \frac{dx}{\sqrt{- (x^2 + 12x + 36 - 28 - 36)}} = \int \frac{dx}{\sqrt{- [(x^2 + 12x + 36) - 28 - 36]}}$$

$$\int \frac{dx}{\sqrt{- [(x^2 + 12x + 36) - 64]}} = \int \frac{dx}{\sqrt{- [(x+6)^2 - 64]}}$$

Se ordena y luego se reemplaza

$$\int \frac{dx}{\sqrt{64 - (x+6)^2}}$$

$$\int \frac{dx}{\sqrt{64 - (x+6)^2}} = \int \frac{8 \cos z dz}{8 \cos z} = \int dz$$

Tabla de integrales

$$\int dz = z + c$$

se reemplaza

$$\int dz = z + c = \arcsen \left(\frac{x+6}{8} \right) + c$$

$$\int \frac{dx}{\sqrt{28 - 12x - x^2}} = \arcsen \left(\frac{x+6}{8} \right) + c$$

$$\begin{aligned}\sqrt{a^2 - x^2} &\Rightarrow x = a \sen z \\ \sqrt{8^2 - (x+6)^2} &\Rightarrow x+6 = 8 \sen z \\ x+6 &= 8 \sen z \\ (x+6)^2 &= 64 \sen^2 z\end{aligned}$$

$$\text{si } x+6 = 8 \sen z \Rightarrow dx = 8 \cos z dz$$

$$\sqrt{64 - (x+6)^2} = \sqrt{64 - (64 \sen^2 z)}$$

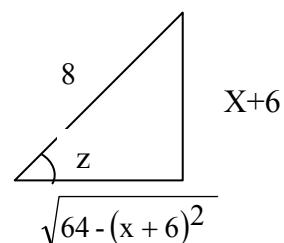
$$\sqrt{64 - (x+6)^2} = \sqrt{64(1 - \sen^2 z)}$$

$$\sqrt{64 - (x+6)^2} = \sqrt{64(\cos^2 z)}$$

$$\sqrt{64 - (x+6)^2} = 8 \cos z$$

$$\text{si } x+6 = 8 \sen z \Rightarrow \sen z \frac{x+6}{8}$$

$$z = \arcsen \left(\frac{x+6}{8} \right)$$



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$$\int \frac{(x+3)dx}{\sqrt{5-4x-x^2}}$$

Se ordena

$$\int \frac{(x+3)dx}{\sqrt{5-4x-x^2}} = \int \frac{(x+3)dx}{\sqrt{-\left(x^2 + 4x - 5\right)}} = \int \frac{(x+3)dx}{\sqrt{-\left(x^2 + 2(2)x - 5\right)}}$$

se completa el cuadrado, sumando 4 y restando 4

$$\int \frac{(x+3)dx}{\sqrt{-\left(x^2 + 4x + 4 - 5 - 4\right)}} = \int \frac{(x+3)dx}{\sqrt{-\left[\left(x^2 + 4x + 4\right) - 5 - 4\right]}}$$

$$\int \frac{(x+3)dx}{\sqrt{-\left[\left(x^2 + 4x + 4\right) - 9\right]}} = \int \frac{(x+3)dx}{\sqrt{-\left[\left(x+2\right)^2 - 9\right]}}$$

Se ordena y luego se reemplaza

$$\int \frac{(x+3)dx}{\sqrt{9 - (x+2)^2}}$$

$$\int \frac{(x+3)dx}{\sqrt{9 - (x+2)^2}} = \int \frac{(x+3)3\cos z dz}{3\cos z} = \int (x+3)dz$$

$$\int (x+3)dz = \int x dz + \int 3 dz$$

Se reemplaza $x = 3 \sin z - 2$

$$\int x dz + \int 3 dz = \int (3 \sin z - 2) dz + 3 \int dz$$

$$\int (3 \sin z - 2) dz + 3 \int dz = \int 3 \sin z - \int 2 dz + 3 \int dz$$

$$3 \int \sin z - 2 \int dz + 3 \int dz$$

Reduciendo términos semejantes

$$3 \int \sin z + \int dz$$

Tabla de integrales

$\int dz = z + c$
$\int \sin z dz = -\cos z + c$

se reemplaza

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{3^2 - (x+2)^2} \Rightarrow x+2 = 3 \sin z$$

$$x+2 = 3 \sin z$$

$$(x+2)^2 = 9 \sin^2 z$$

$$x+2 = 3 \sin z \Rightarrow x = 3 \sin z - 2$$

$$\text{si } x+2 = 3 \sin z \Rightarrow dx = 3 \cos z dz$$

$$\sqrt{9 - (x+2)^2} = \sqrt{9 - (9 \sin^2 z)}$$

$$\sqrt{9 - (x+2)^2} = \sqrt{9(1 - \sin^2 z)}$$

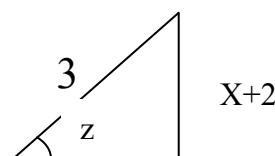
$$\sqrt{9 - (x+2)^2} = \sqrt{9(\cos^2 z)}$$

$$\sqrt{9 - (x+2)^2} = 3 \cos z$$

$$\text{si } x+2 = 3 \sin z \Rightarrow \sin z = \frac{x+2}{3}$$

$$z = \arcsin\left(\frac{x+2}{3}\right)$$

$$\cos z = \frac{\sqrt{9 - (x+2)^2}}{3}$$



$$\sqrt{9 - (x+2)^2}$$

$$3 \int \sin z \, dz + \int dz = -3 \cos z + z + c = -3 \left(\frac{\sqrt{9 - (x+2)^2}}{3} \right) + \arcsin \left(\frac{x+2}{3} \right) + c$$

Simplificando

$$- \sqrt{9 - (x+2)^2} + \arcsin \left(\frac{x+2}{3} \right) + c$$

$$\int \frac{(x+3) \, dx}{\sqrt{5 - 4x - x^2}} = - \sqrt{9 - (x+2)^2} + \arcsin \left(\frac{x+6}{8} \right) + c$$

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$$\int \frac{(2x+3)dx}{9x^2 - 12x + 8} =$$

se completa el cuadrado, sumando 4 y restando 4 para intentar completar el cuadrado

$$\int \frac{(2x+3)dx}{9x^2 - 12x + 8} = \int \frac{(2x+3)dx}{[(3x)^2 - 2(3)(2)x + 8]} = \int \frac{(2x+3)dx}{[(3x)^2 - 12x + 4 + 8 - 4]} = \int \frac{(2x+3)dx}{[(3x)^2 - 12x + 4] + 8 - 4}$$

$$\int \frac{(2x+3)dx}{[(3x)^2 - 12x + 4] + 8 - 4} = \int \frac{(2x+3)dx}{[(3x)^2 - 12x + 4] + 4} = \int \frac{(2x+3)dx}{[(3x-2)^2] + 4} =$$

$$\int \frac{(2x+3)dx}{(3x-2)^2 + 4} =$$

reemplazando

$$\int \frac{(2x+3)dx}{(3x-2)^2 + 4} = \int \frac{(2x+3)\left(\frac{2}{3}\sec^2 z dz\right)}{4\sec^2 z} =$$

$$\int \frac{(2x+3)\left(\frac{2}{3} dz\right)}{4} = \frac{2}{12} \int (2x+3)dz = \frac{1}{6} \int (2x+3)dz$$

$$\frac{1}{6} \int (2x+3)dz = \frac{1}{6} \int 2x dz + \frac{1}{6} \int 3 dz = \frac{1}{3} \int x dz + \frac{1}{2} \int dz$$

$$\text{Pero : } x = \frac{2 \tan z + 2}{3}$$

reemplazando

$$\frac{1}{3} \int x dz + \frac{1}{2} \int dz = \frac{1}{3} \int \left(\frac{2 \tan z + 2}{3} \right) dz + \frac{1}{2} \int dz$$

$$\frac{1}{3} \int \left(\frac{2 \tan z}{3} + \frac{2}{3} \right) dz + \frac{1}{2} \int dz$$

$$\frac{1}{3} \int \frac{2}{3} \tan z dz + \frac{1}{3} \int \frac{2}{3} dz + \frac{1}{2} \int dz$$

$$\frac{2}{9} \int \tan z dz + \frac{2}{9} \int dz + \frac{1}{2} \int dz$$

Sumando términos semejantes

$$a^2 + x^2 \Rightarrow x = a \tan z$$

$$(3x-2)^2 + 2^2 \Rightarrow 3x-2 = 2 \tan z$$

$$3x-2 = 2 \tan z \rightarrow 3x = 2 \tan z + 2 \rightarrow x = \frac{2 \tan z + 2}{3}$$

$$(3x-2)^2 = 4 \tan^2 z$$

$$\text{si } 3x-2 = 2 \tan z \Rightarrow 3 dx = 2 \sec^2 z dz$$

$$dx = \frac{2}{3} \sec^2 z dz$$

$$(3x-2)^2 + 4 = 4 \tan^2 z + 4$$

$$(3x-2)^2 + 4 = 4(\tan^2 z + 1)$$

$$(3x-2)^2 + 4 = 4(\sec^2 z)$$

$$\sec^2 z = \frac{(3x-2)^2 + 4}{4} \Rightarrow \sec z = \frac{\sqrt{(3x-2)^2 + 4}}{2}$$

$$\text{si } 3x-2 = 2 \tan z \Rightarrow \tan z = \frac{3x-2}{2}$$

$$z = \arctan \frac{3x-2}{2}$$

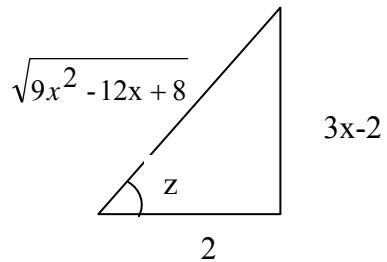
$$\frac{2}{9} \int \operatorname{tg} z \, dz + \frac{13}{18} \int dz$$

Tabla de integrales

$$\int \operatorname{tg} z \, dz = \ln|\sec z| + c$$

$$\int dz = z + c$$

$$\frac{2}{9} \ln|\sec z| + \frac{13}{18} z + c$$



Reemplazando

$$\frac{2}{9} \ln|\sec z| + \frac{13}{18} \arctan \left(\frac{3x-2}{2} \right) + c$$

$$\text{pero } \cos z = \frac{2}{\sqrt{9x^2 - 12x + 8}} \Rightarrow \sec z = \frac{\sqrt{9x^2 - 12x + 8}}{2}$$

$$\frac{2}{9} \ln \left| \frac{\sqrt{9x^2 - 12x + 8}}{2} \right| + \frac{13}{18} \arctan \left(\frac{3x-2}{2} \right) + c$$

$$\frac{2}{(2)^9} \ln \left| \frac{\left(\sqrt{9x^2 - 12x + 8} \right)^2}{(2)^2} \right| + \frac{13}{18} \arctan \left(\frac{3x-2}{2} \right) + c$$

$$\frac{1}{9} \ln \left| \frac{9x^2 - 12x + 8}{4} \right| + \frac{13}{18} \arctan \left(\frac{3x-2}{2} \right) + c$$

$$\frac{1}{9} \ln |9x^2 - 12x + 8| - \frac{1}{9} \ln |4| + \frac{13}{18} \arctan \left(\frac{3x-2}{2} \right) + c$$

$$\text{Pero: } -\frac{1}{9} \ln |4| + c = C_1$$

REEMPLAZANDO

$$\frac{1}{9} \ln |9x^2 - 12x + 8| + \frac{13}{18} \arctan \left(\frac{3x-2}{2} \right) + C_1$$

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$$\int \frac{(x+2)dx}{\sqrt{4x-x^2}}$$

Se factoriza el signo y se ordena

$$\int \frac{(x+2)dx}{\sqrt{-(x^2-4x)}} =$$

se completa el cuadrado, sumando 4 y restando 4

$$\int \frac{(x+2)dx}{\sqrt{-[x^2-4x+4-4]}} = \int \frac{(x+2)dx}{\sqrt{-[(x^2-4x+4)-4]}}$$

$$\int \frac{(x+2)dx}{\sqrt{-[(x-2)^2-4]}}$$

Se ordena, se reemplaza y se simplifica

$$\int \frac{(x+2)dx}{\sqrt{4-(x-2)^2}}$$

$$\int \frac{(x+2)dx}{\sqrt{4-(x-2)^2}} = \int \frac{(x+2)(2\cos z dz)}{2\cos z} = \int (x+2)dz$$

$$\int (x+2)dz = \int x dz + \int 2 dz$$

Se reemplaza $x = 2 \sin z + 2$

$$\int x dz + \int 2 dz = \int (2 \sin z + 2) dz + 2 \int dz$$

$$\int 2 \sin z dz + \int 2 dz + 2 \int dz$$

Reduciendo términos semejantes

$$2 \int \sin z dz + 4 \int dz$$

Tabla de integrales

$\int dz = z + c$
$\int \sin z dz = -\cos z + c$

se reemplaza

$$2 \int \sin z dz + 4 \int dz = -2 \cos z + 4z + c = -2 \left(\frac{\sqrt{4x-x^2}}{2} \right) + 4 \arcsin \left(\frac{x-2}{2} \right) + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{2^2 - (x-2)^2} \Rightarrow x-2 = 2 \sin z$$

$$x-2 = 2 \sin z$$

$$(x-2)^2 = 4 \sin^2 z$$

$$x-2 = 2 \sin z \Rightarrow x = 2 \sin z + 2$$

$$\text{si } x-2 = 2 \sin z \Rightarrow dx = 2 \cos z dz$$

$$\sqrt{4 - (x-2)^2} = \sqrt{4 - (4 \sin^2 z)}$$

$$\sqrt{4 - (x-2)^2} = \sqrt{4(1 - \sin^2 z)}$$

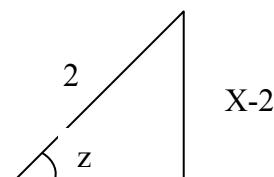
$$\sqrt{4 - (x-2)^2} = \sqrt{4 \cos^2 z}$$

$$\sqrt{4 - (x-2)^2} = 2 \cos z$$

$$\text{si } x-2 = 2 \sin z \Rightarrow \sin z = \frac{x-2}{2}$$

$$z = \arcsin \left(\frac{x-2}{2} \right)$$

$$\cos z = \frac{\sqrt{4x-x^2}}{2}$$



$$\sqrt{4x-x^2}$$

Simplificando

$$-2 \cos z + 4z + c = -\left(\sqrt{4x-x^2}\right) + 4 \arcsen\left(\frac{x-2}{2}\right) + c$$

$$\int \frac{(x+2) dx}{\sqrt{4x-x^2}} = -\sqrt{4x-x^2} + 4 \arcsen\left(\frac{x-2}{2}\right) + c$$

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$$\int \frac{dx}{x^2 - 1}$$

$$\int \frac{dx}{x^2 - 1} = \int \frac{(\sec z \tg z dz)}{\tg^2 z} = \int \frac{\sec z dz}{\tg z}$$

$$\int \sec z \frac{1}{\tg z} dz = \int \frac{1}{\cos z} \ctg z dz = \int \frac{1}{\cos z} \frac{\cos z}{\sin z} dz$$

Simplificando

$$\int \frac{1}{\sin z} dz = \int \csc z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \ctg z| + c$$

$$\int \csc z dz = \ln |\csc z - \ctg z| + c$$

Reemplazando

$$\int \csc z dz = \ln \left| \frac{x}{\sqrt{x^2 - 1}} - \frac{1}{\sqrt{x^2 - 1}} \right| + c$$

$$\int \csc z dz = \ln \left| \frac{x - 1}{\sqrt{x^2 - 1}} \right| + c$$

$$\int \csc z dz = \frac{1}{2} \ln \left| \frac{(x - 1)^2}{(\sqrt{x^2 - 1})^2} \right| + c = \frac{1}{2} \ln \left| \frac{(x - 1)(x - 1)}{x^2 - 1} \right| + c$$

$$\int \csc z dz = \frac{1}{2} \ln \left| \frac{(x - 1)(x - 1)}{(x - 1)(x + 1)} \right| + c$$

$$\int \csc z dz = \frac{1}{2} \ln \left| \frac{(x - 1)}{(x + 1)} \right| + c$$

$$x^2 - a^2 \Rightarrow x = a \sec z$$

$$x^2 - 1 \Rightarrow x = 1 \sec z$$

$$x = \sec z$$

$$X^2 = \sec^2 z$$

$$\text{si } x = \sec z \Rightarrow dx = \sec z \tg z dz$$

$$x^2 - 1 = \sec^2 z - 1$$

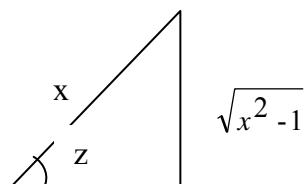
$$x^2 - 1 = \tg^2 z$$

$$x = \sec z$$

$$\text{si } \sec z = x \Rightarrow \cos z = \frac{1}{x}$$

$$\sin z = \frac{\sqrt{x^2 - 1}}{x} \Rightarrow \csc z = \frac{x}{\sqrt{x^2 - 1}}$$

$$\tg z = \sqrt{x^2 - 1} \Rightarrow \ctg z = \frac{1}{\sqrt{x^2 - 1}}$$



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$$\int \frac{dy}{25 - 16y^2} =$$

$$\int \frac{dy}{25 - 16y^2} = \int \frac{\frac{5}{4} \cos z dz}{25 \cos^2 z}$$

Simplificando

$$\frac{5}{100} \int \frac{1}{\cos z} dz = \frac{1}{20} \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \tan z| + c$$

Reemplazando

$$\frac{1}{20} \int \sec z dz = \frac{1}{20} \ln |\sec z + \tan z| + c$$

$$\frac{1}{20} \int \sec z dz = \frac{1}{20} \ln \left| \frac{5}{\sqrt{25 - 16y^2}} + \frac{4y}{\sqrt{25 - 16y^2}} \right| + c$$

$$\frac{1}{20} \int \sec z dz = \frac{1}{20} \ln \left| \frac{5 + 4y}{\sqrt{25 - 16y^2}} \right| + c$$

$$\frac{1}{20} \int \sec z dz = \frac{1}{20(2)} \ln \left| \frac{(5 + 4y)^2}{\left(\sqrt{25 - 16y^2} \right)^2} \right| + c = \frac{1}{40} \ln \left| \frac{(5 + 4y)(5 + 4y)}{25 - 16y^2} \right| + c$$

$$\frac{1}{40} \ln \left| \frac{(5 + 4y)(5 + 4y)}{(5 - 4y)(5 + 4y)} \right| + c = \frac{1}{40} \ln \left| \frac{5 + 4y}{5 - 4y} \right| + c$$

$$\int \frac{dy}{25 - 16y^2} = \frac{1}{40} \ln \left| \frac{5 + 4y}{5 - 4y} \right| + c$$

$$a^2 - y^2 \Rightarrow y = a \sin z$$

$$25 - 16y^2 = 25 - (4y)^2 \Rightarrow 4y = 5 \sin z$$

$$(4y)^2 = 25 \sin^2 z dz$$

$$\text{Si } 4y = 5 \sin z \rightarrow 4 dy = 5 \cos z dz$$

$$dy = \frac{5}{4} \cos z dz$$

$$25 - (4y)^2 = 25 - 25 \sin^2 z$$

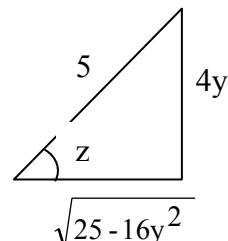
$$25 - (4y)^2 = 25(1 - \sin^2 z)$$

$$25 - (4y)^2 = 25(\cos^2 z)$$

$$\text{si } 4y = 5 \sin z \Rightarrow \sin z = \frac{4y}{5}$$

$$\text{si } \cos z = \frac{\sqrt{25 - 16y^2}}{5} \Rightarrow \sec z = \frac{5}{\sqrt{25 - 16y^2}}$$

$$\tan z = \frac{4y}{\sqrt{25 - 16y^2}}$$



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$$\int \frac{dx}{x^2 + 6x + 8} =$$

Se completa el cuadrado, se adiciona 9 y se le resta 9

$$\int \frac{dx}{x^2 + 6x + 8} = \int \frac{dx}{x^2 + 2(3)x + 9 + 8 - 9} = \int \frac{dx}{(x^2 + 6x + 9) + 8 - 9}$$

$$\int \frac{dx}{(x^2 + 6x + 9) - 1} = \int \frac{dx}{(x+3)^2 - 1}$$

$$\int \frac{dx}{(x+3)^2 - 1} = \int \frac{\sec z \ \tg z \ dz}{\tg^2 z} = \int \frac{\sec z}{\tg z} dz = \int \sec z \frac{1}{\tg z} dz$$

$$\int \frac{1}{\cos z} \ctg z dz = \int \frac{1}{\cos z \ \sin z} \ dz = \int \frac{1}{\sin z} dz = \int \csc z dz$$

Tabla de integrales

$$\int \csc z dz = \ln |\csc z - \ctg z| + c$$

Reemplazando

$$\int \csc z dz = \ln |\csc z - \ctg z| + c$$

$$\ln \left| \frac{x+3}{\sqrt{x^2 + 6x + 8}} - \frac{1}{\sqrt{x^2 + 6x + 8}} \right| + c$$

$$\ln \left| \frac{x+3-1}{\sqrt{x^2 + 6x + 8}} \right| + c = \ln \left| \frac{x+2}{\sqrt{x^2 + 6x + 8}} \right| + c = \frac{1}{2} \ln \left| \frac{(x+2)^2}{(\sqrt{x^2 + 6x + 8})^2} \right| + c$$

$$\frac{1}{2} \ln \left| \frac{(x+2)(x+2)}{(\sqrt{x^2 + 6x + 8})^2} \right| + c = \frac{1}{2} \ln \left| \frac{(x+2)(x+2)}{(x+2)(x+4)} \right| = \frac{1}{2} \ln \left| \frac{x+2}{x+4} \right| + c$$

$$\int \frac{dx}{x^2 + 6x + 8} = \frac{1}{2} \ln \left| \frac{x+2}{x+4} \right| + c$$

$$x^2 - a^2 \Rightarrow x = a \sec z$$

$$(x+3)^2 - 1 \Rightarrow x+3 = 1 \sec z$$

$$x+3 = \sec z$$

$$(X+3)^2 = \sec^2 z$$

$$\text{si } x+3 = \sec z \Rightarrow dx = \sec z \ \tg z \ dz$$

$$(x+3)^2 - 1 = \sec^2 z - 1$$

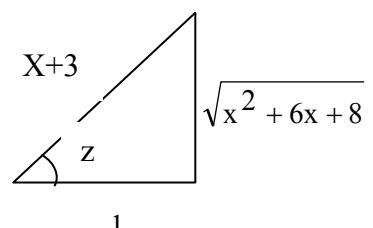
$$(x+3)^2 - 1 = \tg^2 z$$

$$x+3 = \sec z$$

$$\text{si } \sec z = x+3 \Rightarrow \cos z = \frac{1}{x+3}$$

$$\sin z = \frac{\sqrt{x^2 + 6x + 8}}{x+3} \Rightarrow \csc z = \frac{x+3}{\sqrt{x^2 + 6x + 8}}$$

$$\tg z = \sqrt{x^2 + 6x + 8} \Rightarrow \ctg z = \frac{1}{\sqrt{x^2 + 6x + 8}}$$



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$$\int \frac{dx}{4x - x^2} =$$

Se factoriza el signo y se completa el cuadrado, sumando 4 y restando 4.

$$\int \frac{dx}{4x - x^2} = \int \frac{dx}{-x^2 + 2(2)x} = \int \frac{dx}{-x^2 + 4x + 4 - 4} = \int \frac{dx}{-(x^2 - 4x + 4) - 4}$$

$$\int \frac{dx}{-(x-2)^2 - 4} = \int \frac{dx}{4 - (x-2)^2} =$$

$$\int \frac{dx}{4 - (x-2)^2} =$$

reemplazando

$$\int \frac{dx}{4 - (x-2)^2} = \int \frac{2 \cos z dz}{4 \cos^2 z} = \frac{1}{2} \int \frac{1}{\cos z} dz = \frac{1}{2} \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \tan z| + c$$

$$\frac{1}{2} \int \sec z dz = \frac{1}{2} \ln |\sec z + \tan z| + c$$

reemplazando

$$\frac{1}{2} \int \sec z dz = \frac{1}{2} \ln \left| \frac{2}{\sqrt{4x - x^2}} + \frac{x-2}{\sqrt{4x - x^2}} \right| + c$$

$$\frac{1}{2} \int \sec z dz = \frac{1}{2} \ln \left| \frac{2+x-2}{\sqrt{4x - x^2}} \right| + c$$

$$\frac{1}{2} \ln \left| \frac{x}{\sqrt{4x - x^2}} \right| + c = \frac{1}{2(2)} \ln \left| \frac{x^2}{(\sqrt{4x - x^2})^2} \right| + c = \frac{1}{4} \ln \left| \frac{x^2}{4x - x^2} \right| + c$$

$$\frac{1}{4} \ln \left| \frac{x(x)}{(x)(4-x)} \right| + c = \frac{1}{4} \ln \left| \frac{x}{4-x} \right| + c$$

$$\int \frac{dx}{4x - x^2} = \frac{1}{4} \ln \left| \frac{x}{4-x} \right| + c$$

$$a^2 - x^2 \Rightarrow x = a \sin z$$

$$4 - (x-2)^2 = 2^2 - (x-2)^2 \Rightarrow x-2 = 2 \sin z$$

$$\text{Si } x-2 = 2 \sin z \rightarrow dx = 2 \cos z dz$$

$$(x-2)^2 = 4 \sin^2 z$$

$$4 - (x-2)^2 = 4 - 4 \sin^2 z$$

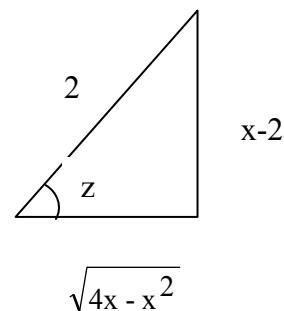
$$4 - (x-2)^2 = 4(1 - \sin^2 z)$$

$$4 - (x-2)^2 = 4(\cos^2 z)$$

$$\text{si } x-2 = 2 \sin z \Rightarrow \sin z = \frac{x-2}{2}$$

$$\text{si } \cos z = \frac{\sqrt{4x - x^2}}{2} \Rightarrow \sec z = \frac{2}{\sqrt{4x - x^2}}$$

$$\tan z = \frac{x-2}{\sqrt{4x - x^2}}$$



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$$\int \frac{dx}{(4-x^2)^{3/2}}$$

$$\int \frac{dx}{(4-x^2)^{3/2}} = \int \frac{2 \cos z \ dz}{8 \cos^3 z}$$

$$\frac{1}{4} \int \frac{1}{\cos^2 z} dz$$

$$= \frac{1}{4} \int \sec^2 z dz$$

Tabla de integrales

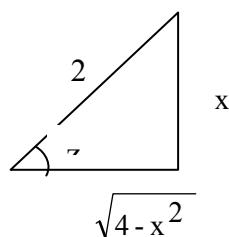
$$\int \sec^2 z dz = \operatorname{tg} z + c$$

$$\frac{1}{4} \int \sec^2 z dz = \frac{1}{4} \operatorname{tg} z + c$$

Reemplazando

$$\frac{1}{4} (\operatorname{tg} z) + c = \frac{1}{4} \left(\frac{x}{\sqrt{4-x^2}} \right) + c$$

$$\int \frac{dx}{(4-x^2)^{3/2}} = \frac{x}{4\sqrt{4-x^2}} + c$$



$$(a^2 - x^2)^{3/2} \Rightarrow x = a \sin z$$

$$(4-x^2)^{3/2} = [(2)^2 - x^2]^{3/2}$$

$$[(2)^2 - x^2]^{3/2} \Rightarrow x = 2 \sin z$$

$$x = 2 \sin z$$

$$x^2 = 4 \sin^2 z$$

$$\text{si } x = 2 \sin z \Rightarrow dx = 2 \cos z dz$$

$$(4-x^2)^{3/2} = [4 - 4 \sin^2 z]^{3/2}$$

$$(4-x^2)^{3/2} = [(4)(1 - \sin^2 z)]^{3/2}$$

$$(4-x^2)^{3/2} = [(4)(\cos^2 z)]^{3/2}$$

$$(4-x^2)^{3/2} = [(4)]^{3/2} * \cos^3 z$$

$$(4-x^2)^{3/2} = \sqrt{64} \cos^3 z$$

$$(4-x^2)^{3/2} = 8 \cos^3 z$$

$$\text{si } x = 2 \sin z \Rightarrow \sin z = \frac{x}{2}$$

$$\operatorname{tg} z = \frac{x}{\sqrt{4-x^2}}$$

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$$\int \frac{\sqrt{25-x^2}}{x} dx$$

$$\int \frac{\sqrt{25-x^2}}{x} dx = \int \frac{5 \cos z (5 \cos z dz)}{5 \sin z}$$

$$\int \frac{5 \cos^2 z}{\sin z} dz$$

Identidad trigonométrica

$$\cos^2 z = 1 - \sin^2 z$$

$$\int \frac{5 \cos^2 z dz}{\sin z} = 5 \int \frac{(1 - \sin^2 z) dz}{\sin z} = 5 \int \frac{1}{\sin z} dz - 5 \int \frac{\sin^2 z}{\sin z} dz$$

$$5 \int \csc z dz - 5 \int \sin z dz$$

Tabla de integrales

$$\begin{aligned}\int \csc z dz &= \ln |\csc z - \cot z| + c \\ \int \sin z dz &= -\cos z + c\end{aligned}$$

$$5 \int \csc z dz - 5 \int \sin z dz = 5 \ln |\csc z - \cot z| - 5(-\cos z) + c$$

$$5 \ln |\csc z - \cot z| + 5 \cos z + c$$

Reemplazando

$$5 \ln \left| \frac{5}{x} - \frac{\sqrt{25-x^2}}{x} \right| + (5) \frac{\sqrt{25-x^2}}{5} + c$$

$$5 \ln \left| \frac{5 - \sqrt{25-x^2}}{x} \right| + \sqrt{25-x^2} + c$$

$$\int \frac{\sqrt{25-x^2}}{x} dx = 5 \ln \left| \frac{5 - \sqrt{25-x^2}}{x} \right| + \sqrt{25-x^2} + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\begin{aligned}\sqrt{5^2 - x^2} &\Rightarrow x = 5 \sin z \\ x^2 &= 25 \sin^2 z\end{aligned}$$

$$\text{si } x = 5 \sin z \Rightarrow dx = 5 \cos z dz$$

$$\sqrt{25 - x^2} = \sqrt{25 - 25 \sin^2 z}$$

$$\sqrt{25 - x^2} = \sqrt{25(1 - \sin^2 z)}$$

$$\sqrt{25 - x^2} = \sqrt{25 \cos^2 z}$$

$$\sqrt{25 - x^2} = 5 \cos z$$

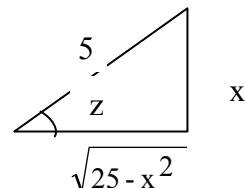
$$\text{si } x = 5 \sin z \Rightarrow \sin z = \frac{x}{5}$$

$$\csc z = \frac{5}{x}$$

$$\tan z = \frac{x}{\sqrt{25-x^2}}$$

$$\cot z = \frac{\sqrt{25-x^2}}{x}$$

$$\cos z = \frac{\sqrt{25-x^2}}{5}$$



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$$\int \sqrt{x^2 + 4} \, dx$$

$$\int \sqrt{x^2 + 4^2} \, dx = \int 2 \sec z \left(2 \sec^2 z \, dz \right) = \int 4 \sec^3 z \, dz$$

$$4 \int \sec^3 z \, dz =$$

Se resuelve la integral $4 \int \sec^3 z \, dz$ **por partes**

$$\int u \, dv = u * v - \int v \, du$$

$$4 \int \sec^3 z \, dz = 4 \int \sec^2 z * \sec z \, dz$$

Se resuelve por partes

$$u = \sec z$$

$$dv = \sec^2 z$$

$$du = \sec z \, \tan z \, dz$$

$$\int dv = \int \sec^2 z \, dz$$

$$v = \tan z$$

$$\int u \, dv = u * v - \int v \, du$$

$$4 \int \sec^2 z * \sec z \, dz = 4 [\sec z * \tan z - \int (\tan z) * \sec z * \tan z \, dz]$$

$$4 \left[\sec z * \tan z - \int \tan^2 z * \sec z \, dz \right]$$

Reemplazando la Identidad trigonométrica
 $\tan^2 z = \sec^2 z - 1$

$$4 \left[\sec z * \tan z - \int (\sec^2 z - 1) * \sec z \, dz \right]$$

$$4 \left[\sec z * \tan z - \int \sec^3 z \, dz + \int \sec z \, dz \right]$$

$$4 \int \sec^3 z \, dz = 4 \left[\sec z * \tan z - \int \sec^3 z \, dz + \int \sec z \, dz \right]$$

$$4 \int \sec^3 z \, dz = 4 \sec z * \tan z - 4 \int \sec^3 z \, dz + 4 \int \sec z \, dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$4 \int \sec^3 z \, dz + 4 \int \sec^3 z \, dz = 4 \sec z * \tan z + 4 \int \sec z \, dz$$

$$8 \int \sec^3 z \, dz = 4 \sec z * \tan z + 4 \int \sec z \, dz$$

Dividiendo la ecuación por 2

$$4 \int \sec^3 z \, dz = 2 \sec z * \tan z + 2 \int \sec z \, dz$$

$$\sqrt{a^2 + x^2} \Rightarrow x = a \tan z$$

$$\sqrt{x^2 + 4} = \sqrt{x^2 + 2^2} \Rightarrow x = 2 \tan z$$

$$x = 2 \tan z$$

$$x^2 = 4 \tan^2 z$$

$$\text{si } x = 2 \tan z \Rightarrow dx = 2 \sec^2 z \, dz$$

$$\sqrt{x^2 + 4} = \sqrt{4 \tan^2 z + 4}$$

$$\sqrt{x^2 + 4} = \sqrt{4(\tan^2 z + 1)}$$

$$\sqrt{x^2 + 4} = \sqrt{4(\sec^2 z)}$$

$$\sqrt{x^2 + 4} = 2 \sec z$$

Tabla de integrales

$$\int \sec z \, dz = \ln |\sec z + \tan z| + c$$

$$4 \int \sec^3 z \, dz = 2 \sec z * \tan z + 2 \ln |\sec z + \tan z| + c$$

$$\int \sqrt{x^2 + 4} \, dx = 2 \sec z * \tan z + 2 \ln |\sec z + \tan z| + c$$

$$\int \sqrt{x^2 + 4} \, dx = 2 \left(\frac{\sqrt{x^2 + 4}}{2} \right) * \left(\frac{x}{2} \right) + 2 \ln \left| \frac{\sqrt{x^2 + 4}}{2} + \frac{x}{2} \right| + c$$

$$\int \sqrt{x^2 + 4} \, dx = 2 \left(\frac{x \sqrt{x^2 + 4}}{4} \right) + 2 \ln \left| \frac{\sqrt{x^2 + 4} + x}{2} \right| + c$$

$$\int \sqrt{x^2 + 4} \, dx = \frac{1}{2} \left(x \sqrt{x^2 + 4} \right) + 2 \ln \left| \frac{\sqrt{x^2 + 4} + x}{2} \right| + c$$

Propiedad de los logaritmos

$$\int \sqrt{x^2 + 4} \, dx = \frac{1}{2} \left(x \sqrt{x^2 + 4} \right) + 2 \ln \left| \sqrt{x^2 + 4} + x \right| - 2 \ln |2| + c$$

$$C_1 = -2 \ln |2| + c$$

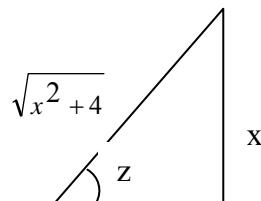
$$x = 2 \tan z$$

$$\text{si } x = 2 \tan z \Rightarrow \tan z = \frac{x}{2}$$

$$\cos z = \frac{2}{\sqrt{a^2 + x^2}}$$

$$\sqrt{x^2 + 4} = 2 \sec z$$

$$\sec z = \frac{\sqrt{x^2 + 4}}{2}$$



2

$$\int \sqrt{x^2 + 4} \, dx = \frac{x}{2} \sqrt{x^2 + 4} + 2 \ln \left| \sqrt{x^2 + 4} + x \right| + C_1$$

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$$\int \sqrt{x^2 - 4} \, dx =$$

$$\int \sqrt{x^2 - 4} \, dx = \int 2 \operatorname{tg} z (2 \sec z \operatorname{tg} z \, dz) = \int 4 \sec z \operatorname{tg}^2 z \, dz$$

$$4 \int \operatorname{tg}^2 z \sec z \, dz =$$

Identidades trigonométricas

$$\operatorname{tg}^2 z = \sec^2 z - 1$$

$$4 \int \operatorname{tg}^2 z \sec z \, dz = 4 \int (\sec^2 z - 1) \sec z \, dz$$

$$4 \int \sec^2 z \sec z \, dz - 4 \int \sec z \, dz$$

$$4 \int \sec^3 z \, dz - 4 \int \sec z \, dz =$$

Integral inicial

Se resuelve la integral $4 \int \sec^3 z \, dz$ **por partes**

$$\int u \, dv = u * v - \int v \, du$$

$$4 \int \sec^3 z \, dz = 4 \int \sec^2 z * \sec z \, dz$$

Se resuelve por partes

$$u = \sec z$$

$$dv = \sec^2 z$$

$$du = \sec z \operatorname{tg} z \, dz$$

$$\int dv = \int \sec^2 z \, dz$$

$$v = \operatorname{ta} z$$

$$\int u \, dv = u * v - \int v \, du$$

$$4 \int \sec^2 z * \sec z \, dz = 4 [\sec z * \operatorname{tg} z - \int (\operatorname{tg} z)^2 * \sec z \, dz]$$

$$4 \left[\sec z * \operatorname{tg} z - \int \operatorname{tg}^2 z * \sec z \, dz \right]$$

Reemplazando la Identidad trigonométrica

$$\operatorname{tg}^2 z = \sec^2 z - 1$$

$$4 \left[\sec z * \operatorname{tg} z - \int (\sec^2 z - 1) * \sec z \, dz \right]$$

$$4 \left[\sec z * \operatorname{tg} z - \int \sec^3 z \, dz + \int \sec z \, dz \right]$$

$$\begin{aligned}\sqrt{x^2 - a^2} &\Rightarrow x = a \sec z \\ \sqrt{x^2 - 2^2} &\Rightarrow x = 2 \sec z\end{aligned}$$

$$x = 2 \sec z$$

$$x^2 = 4 \sec^2 z$$

$$\text{si } x = 2 \sec z \Rightarrow dx = 2 \sec z \operatorname{tg} z \, dz$$

$$\sqrt{x^2 - 2^2} = \sqrt{4 \sec^2 z - 4}$$

$$\sqrt{x^2 - 2^2} = \sqrt{4(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 4} = \sqrt{4(\operatorname{tg}^2 z)}$$

$$\sqrt{x^2 - 4} = 2 \operatorname{tg} z$$

$$4 \int \sec^3 z dz = 4 \left[\sec z * \operatorname{tg} z - \int \sec^3 z dz + \int \sec z dz \right]$$

$$4 \int \sec^3 z dz = 4 \sec z * \operatorname{tg} z - 4 \int \sec^3 z dz + 4 \int \sec z dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$4 \int \sec^3 z dz + 4 \int \sec^3 z dz = 4 \sec z * \operatorname{tg} z + 4 \int \sec z dz$$

$$8 \int \sec^3 z dz = 4 \sec z * \operatorname{tg} z + 4 \int \sec z dz$$

Dividiendo la ecuación por 2

$$4 \int \sec^3 z dz = 2 \sec z * \operatorname{tg} z + 2 \int \sec z dz$$

Tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + c$$

Regresando a la integral inicial después de resolver $4 \int \sec^3 z dz$ por partes.

$$4 \int \sec^3 z dz - 4 \int \sec z dz =$$

$$4 \int \sec^3 z dz - (4 \int \sec z dz) = 2 \sec z * \operatorname{tg} z + 2 \int \sec z dz - (4 \int \sec z dz)$$

$$2 \sec z * \operatorname{tg} z + 2 \int \sec z dz - 4 \int \sec z dz$$

Se reducen términos semejantes

$$2 \sec z * \operatorname{tg} z - 2 \int \sec z dz$$

aplicando la tabla de integrales

$$\int \sec z dz = \ln |\sec z + \operatorname{tg} z| + c$$

$$2 \sec z * \operatorname{tg} z - 2 \ln |\sec z + \operatorname{tg} z| + c$$

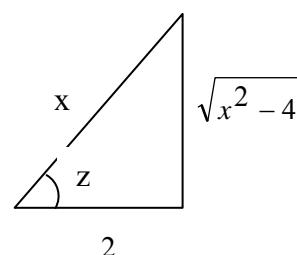
Reemplazando

$$\int \sqrt{x^2 - 4} dz = 2 \sec z * \operatorname{tg} z - 2 \ln |\sec z + \operatorname{tg} z| + c$$

$$\int \sqrt{x^2 - 4} dz = 2 \left(\frac{x}{2} \right) * \left(\frac{\sqrt{x^2 - 4}}{2} \right) - 2 \ln \left| \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2} \right| + c$$

$$\int \sqrt{x^2 - 4} dz = 2 \left(\frac{x \sqrt{x^2 - 4}}{4} \right) - 2 \ln \left| \frac{x + \sqrt{x^2 - 4}}{2} \right| + c$$

$$\begin{aligned} x &= 2 \sec z \\ \text{si } x &= 2 \sec z \Rightarrow \sec z = \frac{x}{2} \\ \cos z &= \frac{2}{x} \\ \sqrt{x^2 - 4} &= 2 \operatorname{tg} z \\ \operatorname{tg} z &= \frac{\sqrt{x^2 - 4}}{2} \end{aligned}$$



$$\int \sqrt{x^2 - 4} dz = \frac{1}{2} x \sqrt{x^2 - 4} - 2 \ln \left| \frac{x + \sqrt{x^2 - 4}}{2} \right| + c$$

Propiedad de los logaritmos

$$\int \sqrt{x^2 - 4} dz = \frac{x}{2} \sqrt{x^2 - 4} - 2 \ln \left| x + \sqrt{x^2 - 4} \right| + 2 \ln |2| + c$$

Pero:

$$C_1 = 2 \ln |2| + c$$

$$\boxed{\int \sqrt{x^2 - 4} dx = \frac{x}{2} \sqrt{x^2 - 4} - 2 \ln \left| x + \sqrt{x^2 - 4} \right| + C_1}$$

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$$\int \frac{x^2 dx}{(4-x^2)^{5/2}}$$

$$\int \frac{x^2 dx}{(4-x^2)^{5/2}} = \int \frac{(4 \sin^2 z)(2 \cos z dz)}{32 \cos^5 z} = \frac{8}{32} \int \frac{\sin^2 z}{\cos^4 z} dz$$

$$\frac{1}{4} \int \frac{\sin^2 z}{\cos^2 z} \frac{1}{\cos^2 z} dz$$

$$\frac{1}{4} \int \tan^2 z \sec^2 z dz =$$

Solución por cambio de variable

$$u = \tan z \quad du = \sec^2 z dz$$

$$\frac{1}{4} \int \tan^2 z \sec^2 z dz = \frac{1}{4} \int (u)^2 du$$

Tabla de integrales

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \Rightarrow n \neq -1$$

$$\frac{1}{4} \int (u)^2 du = \frac{1}{4} \frac{u^3}{3} + C$$

Reemplazando

$$u = \tan z \quad u^3 = \tan^3 z$$

$$\frac{1}{4} \frac{u^3}{3} + C = \frac{1}{12} u^3 + C = \frac{1}{12} (\tan^3 z) + C$$

$$\frac{1}{12} (\tan^3 z) + C = \frac{1}{12} \left(\frac{x}{\sqrt{4-x^2}} \right)^3 + C = \frac{1}{12} \frac{(x)^3}{\left(\sqrt{4x-x^2} \right)^3} + C$$

$$\int \frac{x^2 dx}{(4-x^2)^{5/2}} = \frac{x^3}{12(4x-x^2)^{3/2}} + C$$

$$(a^2 - x^2)^{5/2} \Rightarrow x = a \sin z$$

$$(2^2 - x^2)^{5/2} \Rightarrow x = 2 \sin z$$

$$x^2 = 4 \sin^2 z$$

$$\text{si } x = 2 \sin z \Rightarrow dx = 2 \cos z dz$$

$$(4-x^2)^{5/2} = (4 - 4 \sin^2 z)^{5/2}$$

$$(4-x^2)^{5/2} = [4(1 - \sin^2 z)]^{5/2}$$

$$(4-x^2)^{5/2} = [2^2 (\cos^2 z)]^{5/2}$$

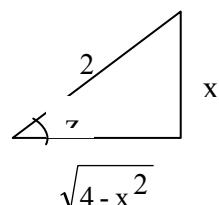
$$(4-x^2)^{5/2} = 2^5 \cos^5 z$$

$$(4-x^2)^{5/2} = 32 \cos^5 z$$

$$\text{si } x = 2 \sin z \Rightarrow \sin z = \frac{x}{2}$$

$$z = \arcsin \frac{x}{2}$$

$$\tan z = \frac{x}{\sqrt{4-x^2}}$$



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$$\int \frac{dx}{x^2 \sqrt{9-x^2}}$$

$$\int \frac{dx}{x^2 \sqrt{9-x^2}} = \int \frac{3 \cos z \ dz}{9 \sin^2 z (3 \cos z)}$$

$$\frac{1}{9} \int \frac{1}{\sin^2 z} dz$$

$$\frac{1}{9} \int \csc^2 z dz$$

Tabla de integrales

$$\int \csc^2 z dz = -\operatorname{ctg} z + c$$

$$\frac{1}{9} \int \csc^2 z dz = -\frac{1}{9} \operatorname{ctg} z + c$$

Reemplazando

$$-\frac{1}{9} (\operatorname{ctg} z) + c = -\frac{1}{9} \left(\frac{\sqrt{9-x^2}}{x} \right) + c$$

$$\int \frac{dx}{x^2 \sqrt{9-x^2}} = -\frac{\sqrt{9-x^2}}{9x} + c$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$\sqrt{3^2 - x^2} \Rightarrow x = 3 \sin z$$

$$x = 3 \sin z$$

$$x^2 = 9 \sin^2 z$$

$$\text{si } x = 3 \sin z \Rightarrow dx = 3 \cos z dz$$

$$\sqrt{9-x^2} = \sqrt{9 - 9 \sin^2 z}$$

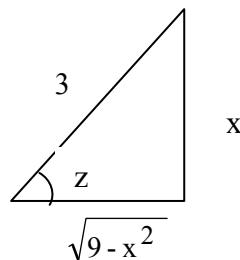
$$\sqrt{9-x^2} = \sqrt{9 (1 - \sin^2 z)}$$

$$\sqrt{9-x^2} = \sqrt{9 \cos^2 z}$$

$$\sqrt{9-x^2} = 3 \cos z$$

$$\text{si } x = 3 \sin z \Rightarrow \sin z = \frac{x}{3}$$

$$\operatorname{tg} z = \frac{x}{\sqrt{9-x^2}} \quad \operatorname{ctg} z = \frac{\sqrt{9-x^2}}{x}$$



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$$\int \frac{x^2 dx}{\sqrt{x^2 - 16}}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 - 16}} = \int \frac{(16 \sec^2 z) 4 \sec z \tan z dz}{(4 \tan z)}$$

$$16 \int \sec^3 z dz =$$

Se resuelve la integral $16 \int \sec^3 z dz$ **por partes**

$$\int u dv = u * v - \int v du$$

$$16 \int \sec^3 z dz = 16 \int \sec^2 z * \sec z dz$$

Se resuelve por partes

$$u = \sec z$$

$$dv = \sec^2 z$$

$$du = \sec z \tan z dz$$

$$\int dv = \int \sec^2 z dz$$

$$v = \tan z$$

$$16 \int \sec^2 z * \sec z dz = 16 [\sec z * \tan z - \int (\tan z)^2 * \sec z dz]$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec z$$

$$\sqrt{x^2 - 16} = \sqrt{x^2 - 4^2}$$

$$\sqrt{x^2 - 4^2} \Rightarrow x = 4 \sec z$$

$$x = 4 \sec z$$

$$x^2 = 16 \sec^2 z$$

$$\text{si } x = 4 \sec z \Rightarrow dx = 4 \sec z \tan z dz$$

$$\sqrt{x^2 - 16} = \sqrt{16 \sec^2 z - 16}$$

$$\sqrt{x^2 - 16} = \sqrt{16(\sec^2 z - 1)}$$

$$\sqrt{x^2 - 16} = \sqrt{16(\tan^2 z)}$$

$$\sqrt{x^2 - 16} = 4 \tan z$$

$$16 [\sec z * \tan z - \int (\tan z)^2 * \sec z dz] = 16 [\sec z * \tan z - \int \tan^2 z * \sec z dz]$$

Reemplazando la Identidad trigonométrica $\tan^2 z = \sec^2 z - 1$

$$16 [\sec z * \tan z - \int \tan^2 z * \sec z dz] = 16 [\sec z * \tan z - \int (\sec^2 z - 1) * \sec z dz]$$

$$16 [\sec z * \tan z - \int \sec^3 z dz + \int \sec z dz]$$

$$16 \int \sec^3 z dz = 16 [\sec z * \tan z - \int \sec^3 z dz + \int \sec z dz]$$

$$16 \int \sec^3 z dz = 16 \sec z * \tan z - 16 \int \sec^3 z dz + 16 \int \sec z dz$$

Ordenando como una ecuación cualquiera y simplificando los términos semejantes

$$16 \int \sec^3 z dz + 16 \int \sec^3 z dz = 16 \sec z * \tan z + 16 \int \sec z dz$$

$$32 \int \sec^3 z dz = 16 \sec z * \tan z + 16 \int \sec z dz$$

Dividiendo la ecuación por 2

$$16 \int \sec^3 z dz = 8 \sec z * \tan z + 8 \int \sec z dz$$

Tabla de integrales

$$\int \sec z \, dz = \ln |\sec z + \tan z| + c$$

$$16 \int \sec^3 z \, dz = 8 \sec z * \tan z + 8 \ln |\sec z + \tan z| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 16}} = 8 \sec z * \tan z + 8 \ln |\sec z + \tan z| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 16}} = 8 \left(\frac{x}{4} \right) * \left(\frac{\sqrt{x^2 - 16}}{4} \right) + 8 \ln \left| \frac{x}{4} + \frac{\sqrt{x^2 - 16}}{4} \right| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 16}} = 8 \left(\frac{x \sqrt{x^2 - 16}}{16} \right) + 8 \ln \left| \frac{x + \sqrt{x^2 - 16}}{4} \right| + c$$

$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 16}} = \left(\frac{x \sqrt{x^2 - 16}}{2} \right) + 8 \ln |x + \sqrt{x^2 - 16}| - 8 \ln |4| + c$$

Pero: $C_1 = -8 \ln |4| + c$

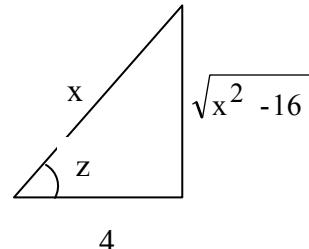
$$\int \frac{x^2 \, dx}{\sqrt{x^2 - 16}} = \left(\frac{x \sqrt{x^2 - 16}}{2} \right) + 8 \ln |x + \sqrt{x^2 - 16}| + c_1$$

$$x = 4 \sec z$$

$$\text{si } x = 4 \sec z \Rightarrow \sec z = \frac{x}{4}$$

$$\cos z = \frac{4}{x}$$

$$\tan z = \frac{\sqrt{x^2 - 16}}{4}$$



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$$\int x^3 \sqrt{a^2 - x^2} dx$$

$$\int x^3 \sqrt{a^2 - x^2} dx = \int (a^3 \sin^3 z)(a \cos z)(a \cos z dz)$$

$$a^5 \int \sin^3 z \cos^2 z dz = a^5 \int (\sin^2 z) \sin z \cos^2 z dz$$

Reemplazando la Identidad trigonométrica

$$\sin^2 z = 1 - \cos^2 z$$

$$a^5 \int (1 - \cos^2 z) \sin z \cos^2 z dz = a^5 \int (1 - \cos^2 z) \cos^2 z \sin z dz$$

Solución por cambio de variable

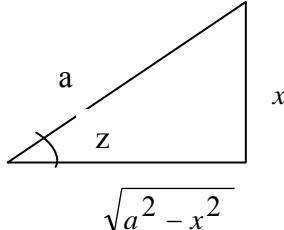
$$u = \cos z \quad du = -\sin z dz$$

$$-a^5 \int (1 - u^2) u^2 du = -a^5 \int u^2 du + a^5 \int u^4 du$$

Tabla de integrales

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \Rightarrow n \neq -1$$

$$-\frac{a^5 u^3}{3} + \frac{a^5 u^5}{5} + C$$



Reemplazando

$$u = \cos z \quad u^3 = \cos^3 z \quad u^5 = \cos^5 z$$

$$-\frac{a^5 u^3}{3} + \frac{a^5 u^5}{5} + C = -\frac{a^5 (\cos^3 z)}{3} + \frac{a^5 (\cos^5 z)}{5}$$

$$-\frac{a^5 (\cos z)^3}{3} + \frac{a^5 (\cos z)^5}{5} = -\frac{a^5}{3} \left(\frac{\sqrt{a^2 - x^2}}{a} \right)^3 + \frac{a^5}{5} \left(\frac{\sqrt{a^2 - x^2}}{a} \right)^5 + C = -\frac{a^5}{3} \frac{\left(\sqrt{a^2 - x^2} \right)^3}{a^3} + \frac{a^5}{5} \frac{\left(\sqrt{a^2 - x^2} \right)^5}{a^5} + C$$

$$\int x^3 \sqrt{a^2 - x^2} dx = -\frac{a^2}{3} (a^2 - x^2)^{3/2} + \frac{1}{5} (a^2 - x^2)^{5/2} + C$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin z$$

$$x^2 = a^2 \sin^2 z \Rightarrow x^3 = a^3 \sin^3 z$$

$$\text{Si } x = a \sin z \rightarrow dx = a \cos z dz$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2 z}$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 (1 - \sin^2 z)}$$

$$\sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2 z}$$

$$\sqrt{a^2 - x^2} = a \cos z$$

$$\text{si } x = a \sin z \Rightarrow \sin z = \frac{x}{a}$$

$$\cos z = \frac{\sqrt{a^2 - x^2}}{a}$$