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COORDINACIÓN DE INGENIERÍA DE TELECOMUNICACIONES

Ejercicios de Transformadas

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Ejercicios de Transformadas.

Fracciones Parciales:

- ✓ **$1 / s (s^2 + 2) =$** $A/ s + Bs + C / s^2 + 2$

$1= A (s^2 + 2) + s (Bs + c)$
$1= As^2 + 2A + Bs^2 + Cs$
$1= (A+B) s^2 + Cs + 2A$

$A+B =0$
$C=0$
$2A=1$
$½ + B = 0$

A = 1 /2	B = -1 /2	C= 0

$1 / s (s^2 + 2) =$ **$½ / s – ½ s + 0 / s^2 + 2$**

- ✓ **$3 – s / s (s^2 +1) =$** $A/s + Bs+ C / s^2 + 1$

$3 - s = A (s^2 + 1) + s (Bs +C) / s^2 + 1$
$3 - s= As^2 + A + Bs^2 + Cs$
$3 - s= (A+B) s^2 + Cs + A$

$A+B =0$
$C= -1$
$A=3$
$3 + B = 0$

A = 3	B= -3	C = - 1

$3 – s / s (s^2 +1) =$ **$3/s - 3s -1 / s^2 + 1$**

- ✓ **$3s +1 / s^2 (s^2 + 25) =$** $As + B / s^2 + Cs +D / s^2 +25$

$3 s + 1= (s^2 +25)(As+ B) + s^2 (Cs +D)$

$3\ s + 1 = s^2A + Bs^2 + 25As + 25B + s^2C + Ds^2$
$3\ s + 1 = (A+C)\ s^3 + (B+D)s^2 + 25As + 25\ B$

A+C = 0
B+D= 0
25A= 3
25B= 1

A = 3 / 25	B = 1 /25

A+C=0 sustituyendo el valor de A queda:
3/ 25 + C= 0

C = -3 /25

B + D= 0 sustituyendo el valor de B queda
1 /25 + D=0

D = -1 /25

$3s + 1 / s^2 (s^2 + 25) =$ **<u>$3/25\ s + 1/25 / s^2 - 3/25s - 1/25 / s^2 + 25$</u>**

- ✓ **$1 / s^4 – 16$** $= A / (s - 2) + B/ (s+2) + Cs + D / s^4 - 16$

$1 = A (s+2)(s^4 – 4) + B (s-2)(s^2 - 4) + (Cs + D)(s – 2)(s + 2)$

<u>Cuando s=2</u>

1= A(4)(8) + 0+ 0
1= 32 A
A = 1/32

Cuando s = -2

1 = 0+ B (-4)(8) + 0
1= -32 A
B= -1/32

Desarrollando:

$1 = (As + 2\ A)(s^4 + 4) + (Bs - 2B)(s^4 + 4) + (Cs + D)\ (s^2 – 4)$
$1 = As^3 + 4As + 2\ As^2 + 8\ A + Bs^3 + 4Bs - 2Bs^2 - 8B + Cs^3 – 4Cs + Ds^2 – 4D$
$1 = (A+B+C)\ s^3 + (2\ A - 2B + D)s^2 + (4\ A + 4B - 4C)\ s + (8\ A – 8B – 4D)$

1. A+B+C=0
2. 2 A - 2B +D=0
3. 4 A +4B – 4C =0
4. 8 A – 8B – 4D=1

De la 1 sustituimos:

1/32 - 1/32 + C = 0

C = 0

2* 1/32 – 2(-1/32) + D =0
2 /32 + 2/32 + D= 0
4/32 + D =0

D = -4 /32

1 / s^4 – 16 = **<u>1/32 / (s - 2) – 1/32/ (s+2) - 4/32 / s^4 -16</u>**

Números Complejos:
Parte A

- ✓ **5 i / 2+ i =** 1+ 2i

5 i / 2+ i * 2 – i / 2 – i = (10i + 5 / 4+1) =(10i + 5 / 5) = 10i /5 + 5/ 5 = 2i + 1 =**<u>1+2i</u>**

- ✓ **$(-1+i)^7$ =** - 8(1 +i)

[- 1 +i] = $\sqrt{(1)^2 + (1)^2}$ = $\sqrt{2}$

θ= arctg $^{-1}$ 1 = phi /4
θ= phi – phi /4 = 3phi /4

$(-1+i)^7$ = $\sqrt{2}$ (cos (3phi/4) + i sen (3phi /4)) ^{7}
$(-1+i)^7$ = $\sqrt{(2)^7}$ (cos (3phi/4)*7 + i sen (3phi /4)*7)
$(-1+i)^7$ = 8 $\sqrt{2}$ (cos (21phi/4) + i sen (21phi /4)*7)
$(-1+i)^7$ = 8 $\sqrt{2}$ (- cos (21phi/4) - i sen (21phi /4))

$(-1+i)^7$ = 8 $\sqrt{2}$ (-$\sqrt{2}$ / 2 – i$\sqrt{2}$ / 2)
$(-1+i)^7$ = 8 ($\sqrt{4}$ / 2 + i$\sqrt{4}$ / 2)
$(-1+i)^7$ = 8 (2 /2 + i 2 /2)
$(-1+i)^7$ = **8 (1 +i)**

- ✓ **$i (1 - i \sqrt{3}) (\sqrt{3} + 1)$** $= 2 + 2 i \sqrt{3}$

  $i (1 + i \sqrt{3}) (\sqrt{3} + 1)$

  $i \sqrt{3} + i^2 + \sqrt{9} + i \sqrt{3}$

  $i \sqrt{3} - 1 + 3 + i \sqrt{2}$

  $\underline{2 + 2 i \sqrt{3}}$

- ✓ **$(1 + i \sqrt{3})^{-10} – 2^{-11} =$**$(- 1 + i \sqrt{3})$

  $(1 + i \sqrt{3})^{-10} = 1 / (1 + i \sqrt{3})^{10}$

  $[(1 + i \sqrt{3})] = \sqrt{1 + 3} = \sqrt{4} = 2$

  $\theta = \text{arctg}^{-1} \sqrt{3}$ = phi / 3

  $(1 + i \sqrt{3})^{-10} = 1/ (2 \text{ cis phi } /3)^{10} = 1 / (2^{10} \text{ cis } 10\text{phi} / 3)$

  $1 / 2^{10}$ (cos phi / 3 + i sen phi / 3)

  $1 / (2^{10} (-1/2 + i \sqrt{3} /2))$

  $1/ (2^{10} (-1/2 + i \sqrt{3} /2) * (-1/2 + i \sqrt{3} /2) / (-1/2 + i \sqrt{3} /2))$

  $1 / (2^{10} (1/4 + 3/4) * (-1/2 + i \sqrt{3} /2) / 2^{10} * 1)$

  $(1 /2) * 2^{10} (-1/2 + i \sqrt{3} /2) / 1 = (2^{-10} /2) * (-1/2 + i \sqrt{3} /2) =$ **$2^{-11} (-1 + i \sqrt{3})$**

Parte B

- ✓ [z-1-i] = 1

[(x+iy) – 1 – i] = 1
[x- 1 +i(y – 1] = 1
$\sqrt{(x-1)^2+(y-1)^2}$ = 1
$(x-1)^2 + (y-1)^2$ = **1**

- ✓ [z – 1] = [z +i]

[(x +iy) – 1] = [(x+iy) +i]
[(x – 1) + iy] = [(x+i) (y+1)]
$\sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y+1)^2}$
X^2 - 2x +1 + y^2 = x^2 + y^2 +2y+1
-2x +1 = 2y + 1
-2x -2y +1 -1 = 0
-2x – 2y = 0
x=y

- ✓ [z – 1+ i] >/ [z -1- i]

[x+iy -1 +i] >/ [x+iy -1- i]
[(x-1) +i (y +1)] = [(x-1) i (y-1)]
$\sqrt{(x-1)^2+(y+1)^2}$ >/ $\sqrt{(x-1)^2+(y-1)^2}$
$(y+1)^2$ >/ $(y-1)^2$
y^2 + 2y +1 >/ y^2 - 2y +1
2y >/ - 2y
2y + 2y >/ 0
4y >/ 0
y>/ 0

- ✓ Cos (z –i) = 2

Cos^{-1} (2)= z-1
Z = Cos^{-1} (2) + i

Por formula:

Cos^{-1} (z) = [-i log (z + i $(1-z^2)^{½}$]
Z = [-i log (2 + i $(1-z^2)^{½}$] +i + 2phi* n
Z = [i -i log (2 + i $(-3^2)^{½}$] +i + 2phi* n
Z = [i -i log (2 + i $(\sqrt{3}\,i)^{½}$] + 2phi n
Z = [i -i log (2 +/- $(\sqrt{3})^{½}$] + 2phi n
Z= **2phi*n + i [1- log (2 +/- ($\sqrt{3}$)**

- ✓ Encontrar la ecuación de la circunferencia que pasa por los puntos z1=1-i, z2=2i y z3= 1+i

La ecuación de la circunferencia es la siguiente:
$X^2 + y^2$ +Dx + Ey + F=0

Los números complejos en su forma par:
Z1= (1,-1) = 1+1 +D + E+ F=0
Z2= (0,2) = 0+ 4+ 0+2E +F =0
Z3= (1,1) = 1+1+D + E+F =0

D+ E+ F = - 2
2E + F = - 4
D+ E+ F = - 2

Resolver por sistema de Gauss, el resultado es el siguiente:

$X^2 + y^2 + 2x - 4 = 0$
$(X^2 + 2x) + y^2 = 4$
$(X^2 + 2x + 1) + y^2 = 4 + 1$
$(X^2 + 1)\ y^2 = 5$