ESTATICA

CAPITULO 9

CENTROS DE GRAVEDAD y CENTROIDE

HIBBELER

Edición 10

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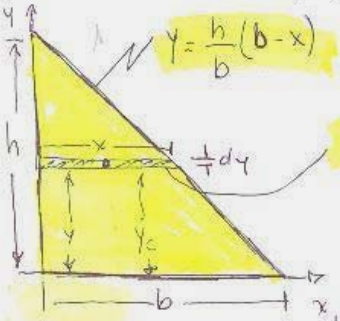
Determine la distancia $\bar{Y}$ del eje "x" al centroide del area del triangulo mostrado en la figura.



Se halla el area por diferencial horizontal

NOTA: De antemano se sabe que el area de un triangulo es $A = \frac{b\,h}{2}$

$$A = \int dA = \int_0^h x.dy$$

$$A = \int_0^h \left(b - \frac{y}{h}b\right)dy$$

$$A = \int_0^h b\,dy - \int_0^h \frac{y}{h}b\,dy$$

$$A = b\int_0^h dy - \frac{b}{h}\int_0^h y\,dy$$

$$A = b(y)\Big|_0^h - \frac{b}{h}\frac{(y)^2}{2}\Big|_0^h$$

$$A = b(y)\Big|_0^h - \frac{b(y)^2}{2h}\Big|_0^h$$

$$A = b(h) - b(0) - \frac{b}{2h}(h)^2 + \frac{b}{2h}(0)^2$$

$$A = bh - \frac{1}{2}bh =$$

$$A = \frac{2bh - bh}{2} = \frac{1}{2}b.h$$

$$\boxed{A = \frac{1}{2}bh}$$ Area total del triangulo

Pero

$A = x.dy$

$y = \frac{h}{b}(b-x)$

Despejamos "x"

$yb = h(b-x)$

$\frac{yb}{h} = b - x$

$$\boxed{x = b - \frac{yb}{h}}$$

Se halla $\int y_c\,dA$

$$\int y_c\,dA = \int_0^h y(x\,dy)$$

Pero $y = y_c$

$A = x\,dy$

$$\int y_c\,dA = \int_0^h y\left(b - \frac{b}{h}y\right)dy$$

Pero:

$$\boxed{x = b - \frac{b}{h}.y}$$

$$\int y_c\,dA = \int_0^h by\,dy - \int_0^h \frac{b}{h}y^2dy$$

$$\int y_c\,dA = b\int_0^h y\,dy - \frac{b}{h}\int_0^h y^2dy$$

$$\int y_c\,dA = b\frac{(y)^2}{2}\Big|_0^h - \frac{b}{h}\frac{(y)^3}{3}\Big|_0^h$$

$$\int y_c\,dA = \frac{b}{2}(h)^2 - \frac{b}{2}(0)^2 - \frac{b}{3h}(h)^3 + \frac{b}{3h}(0)^3$$

$$\int y_c\,dA = \frac{1}{2}bh^2 - \frac{bh^3}{3h}$$

$$\int y_c\,dA = \frac{1}{2}bh^2 - \frac{b}{3}h^2$$

$$\int y_c\,dA = \frac{3bh^2 - 2bh^2}{6} = \frac{bh^2}{6}$$

$$\boxed{\int y_c\,dA = \frac{bh^2}{6}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{bh^2}{6}}{\frac{bh}{2}} = \frac{2bh^2}{6bh}$$

$$\boxed{\bar{y} = \frac{1}{3}h}$$

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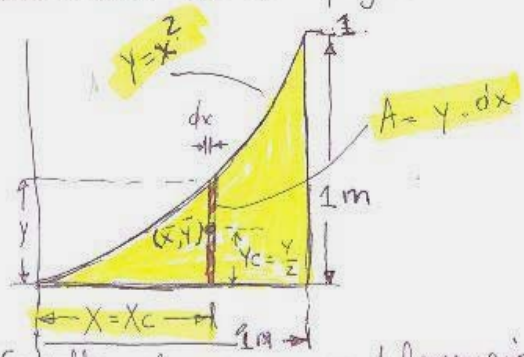
Localice el centroide del area mostrada en la figura.



Se halla el area por diferencial vertical

$$A = \int dA = \int_0^1 y\,dx$$

Pero

$A = y\,dx$

$y = x^2$

$$A = \int_0^1 x^2\,dx$$

$$A = \frac{(x)^3}{3}\Big|_0^1 = \frac{(1)^3}{3} - \frac{(0)^3}{3}$$

$$A = \frac{1}{3}$$

AREA TOTAL

Hallar $\int y_c\,dA =$

$$\int y_c\,dA = \int_0^1 \left(\frac{y}{2}\right)(y\,dx)$$

$$\int y_c\,dA = \frac{1}{2}\int_0^1 y^2\,dx = \frac{1}{2}\int_0^1 x^4\,dx$$

Pero

$y_c = \frac{y}{2}$

$dA = y\,dx$

Pero $y = x^2$

$y^2 = x^4$

$$\int y_c\,dA = \frac{1}{2}\frac{(x)^5}{5}\Big|_0^1 = \frac{1}{10}x^5\Big|_0^1$$

$$\int y_c\,dA = \frac{1}{10}(1)^5 - \frac{1}{10}(0)^5$$

$$\int y_c\,dA = \frac{1}{10}$$

Hallar

$$\int x_c\,dA$$

$$\int x_c\,dA = \int_0^1 (x)(y\,dx)$$

Pero

$X = X_c$

$dA = y\,dx$

Pero

$y = x^2$

$$\int x_c\,dA = \int_0^1 (x)(x^2)\,dx$$

$$\int x_c\,dA = \int_0^1 x^3\,dx$$

$$\int x_c\,dA = \frac{x^4}{4}\Big|_0^1$$

$$\int x_c\,dA = \frac{(1)^4}{4} - \frac{(0)^4}{4} = \frac{1}{4}$$

$$\int x_c\,dA = \frac{1}{4}$$

$$\bar{X} = \frac{\int x_c\,dA}{A} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

$$\bar{X} = \frac{3}{4}$$

$$\bar{Y} = \frac{\int y_c\,dA}{A} = \frac{\frac{1}{10}}{\frac{1}{3}} = \frac{3}{10}$$

$$\bar{Y} = \frac{3}{10}$$

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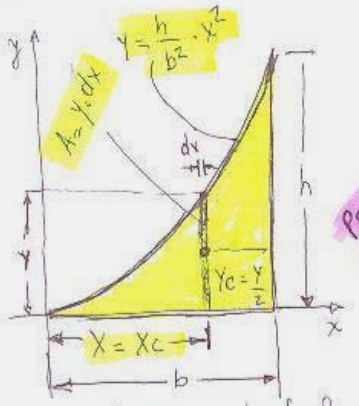
Localice el centroide del area sombreada

PROBLEMA 9.9 HIBBELER EDIC 10



Se halla el area total sombreada por diferencial vertical.

$$A=\int dA=\int_0^b y\,dx$$

$$A=\int_0^b \left(\frac{h}{b^2}x^2\right)dx$$

$$A=\frac{h}{b^2}\int_0^b x^2\,dx$$

$$A=\frac{h}{b^2}\frac{(x)^3}{3}\Big|_0^b=\frac{h}{3b^2}(b)^3-\frac{h}{3b^2}(0)^3$$

$$A=\frac{hb^3}{3b^2}=\frac{hb}{3}$$

$$A=\frac{hb}{3}$$ Area sombreada

Pero

$A=y\,dx$

$y=\frac{h}{b^2}\cdot x^2$

$$\int X_c\,dA=$$

$$\int X_c\,dA=\int_0^b x(y\,dx)$$

$$\int X_c\,dA=\int_0^b x\left(\frac{h}{b^2}x^2\right)dx$$

$$\int X_c\,dA=\frac{h}{b^2}\int_0^b x^3\,dx=\frac{h}{b^2}\frac{(x)^4}{4}\Big|_0^b$$

$$\int X_c\,dA=\frac{h(b)^4}{b^2\,4}-\frac{h}{b^2}\frac{(0)^4}{4}=\frac{hb^4}{4b^2}$$

Pero

$X=X_c$

$dA=y\,dx$

$y=\frac{h}{b^2}\cdot x^2$

$$\int X_c\,dA=\frac{hb^2}{4}$$

hallar $\int Y_c\,dA$

$$\int Y_c\,dA=\int_0^b\left(\frac{y}{2}\right)(y\,dx)$$

$$\int Y_c\,dA=\frac{1}{2}\int_0^b y^2\,dx$$

$$\int Y_c\,dA=\frac{1}{2}\int_0^b\left(\frac{h^2}{b^4}x^4\right)dx$$

$$\int Y_c\,dA=\frac{h^2}{2b^4}\int_0^b x^4\,dx$$

$$\int Y_c\,dA=\frac{h^2}{2b^4}\frac{(x)^5}{5}\Big|_0^b$$

$$\int Y_c\,dA=\frac{h^2}{2b^4}\frac{(b)^5}{5}-\frac{h^2}{2b^4}\frac{(0)^5}{5}$$

$$\int Y_c\,dA=\frac{h^2b^5}{10b^4}=\frac{h^2b}{10}$$

$$\int Y_c\,dA=\frac{h^2b}{10}$$

Pero

$A=y\,dx$

$Y_c=\frac{y}{2}$

$y=\frac{h}{b^2}x^2$

$$y^2=\frac{h^2}{b^4}\cdot x^4$$

$$\bar{x}=\frac{\int x_c\,dA}{A}=\frac{\frac{hb^2}{4}}{\frac{hb}{3}}=\frac{3hb^2}{4hb}=\frac{3}{4}b$$

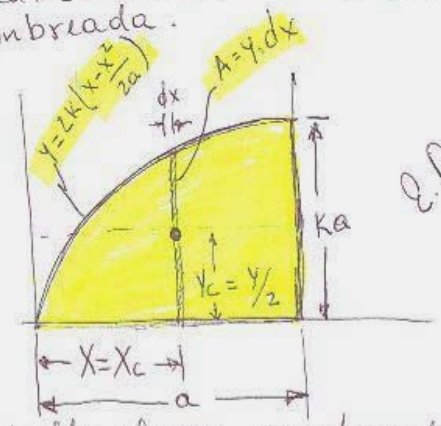
$$\bar{x}=\frac{3}{4}b$$

$$\bar{y}=\frac{\int Y_c\,dA}{A}=\frac{\frac{h^2b}{10}}{\frac{hb}{3}}=\frac{3h^2b}{10hb}=\frac{3}{10}h$$

$$\bar{y}=\frac{3}{10}h$$

Problema 9.10 Hibbeler edic 10

Localice el centroide $\bar{x}$ del area sombreada.



E. Quintero

Se halla el area sombreada por diferencial vertical

$$A=\int dA=\int_0^a y.dx$$

$$A=\int_0^a\left[2k\left(x-\frac{x^2}{2a}\right)\right]dx$$

$$A=2k\int_0^a\left(x-\frac{x^2}{2a}\right)dx$$

$$A=2k\int_0^a x\,dx-2k\int\frac{x^2}{2a}dx$$

$$A=2k\int_0^a x\,dx-\frac{2k}{2a}\int x^2dx$$

$$A=2k\frac{(x)^2}{2}\Big|_0^a-\frac{k}{a}\frac{(x)^3}{3}\Big|_0^a$$

$$A=kx^2\Big|_0^a-\frac{k(x)^3}{3a}\Big|_0^a$$

$$A=k(a)^2-k(0)^2-\frac{k}{3a}(a)^3+\frac{k}{3a}(0)^3$$

$$A=ka^2-\frac{ka^3}{3a}=ka^2-\frac{ka^2}{3}$$

$$A=\frac{3ka^2-ka^2}{3}=\frac{2ka^2}{3}$$

$$\boxed{A=\frac{2ka^2}{3}}$$ Area sombreada

Pero

$A=y.dx$

$y=2k\left(x-\frac{x^2}{2a}\right)$

Hallar $\int X_c\,dA$

$$\int X_c\,dA=\int_0^a \bar{X}(y\,dx)$$

Pero $X=X_c$

$dA=y\,dx$

$y=2k\left(x-\frac{x^2}{2a}\right)$

$$\int X_c\,dA=\int_0^a x\left[2k\left(x-\frac{x^2}{2a}\right)\right]dx$$

$$\int X_c\,dA=2k\int_0^a x\left[x-\frac{x^2}{2a}\right]dx$$

$$\int X_c\,dA=2k\int_0^a\left(x^2-\frac{x^3}{2a}\right)dx$$

$$\int X_c\,dA=2k\int_0^a x^2dx-2k\int_0^a\frac{x^3}{2a}dx$$

$$\int X_c\,dA=2k\int_0^a x^2dx-\frac{2k}{2a}\int_0^a x^3dx$$

$$\int X_c\,dA=2k\int_0^a x^2dx-\frac{k}{a}\int_0^a x^3dx$$

$$\int X_c\,dA=2k\frac{(x)^3}{3}\Big|_0^a-\frac{k}{a}\frac{(x)^4}{4}\Big|_0^a$$

$$\int X_c\,dA=\frac{2k}{3}(a)^3-\frac{2k}{3}(0)^3-\frac{k}{4a}(a)^4+\frac{k}{4a}(0)^4$$

$$\int X_c\,dA=\frac{2ka^3}{3}-\frac{ka^4}{4a}=\frac{2ka^3}{3}-\frac{ka^3}{4}$$

$$\int X_c\,dA=\frac{8ka^3-3ka^3}{12}=\frac{5ka^3}{12}$$

$$\boxed{\int X_c\,dA=\frac{5ka^3}{12}}$$

$$\bar{X}=\frac{\int X_c\,dA}{A}=\frac{\frac{5ka^3}{12}}{\frac{2ka^2}{3}}=\frac{3(5ka^3)}{12(2ka^2)}$$

$$\bar{X}=\frac{15a}{24}=\frac{5a}{8}$$

$$\boxed{\bar{X}=\frac{5a}{8}}$$