PROBLEMAS RESUELTOS DE ESTRUCTURAS EN EQUILIBRIO

CAP 6 ESTATICA BEDFORD
6.1 Armaduras
6.2 Método de las juntas o nudos

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Método de las juntas o nudos
El método de las juntas implica dibujar diagramas de cuerpo libre de las juntas de una armadura, una por una, y usar las ecuaciones de equilibrio para determinar las fuerzas axiales en las barras. Por lo general, antes debemos dibujar un diagrama de toda la armadura (es decir, tratar la armadura como un solo cuerpo) y calcular las reacciones en sus soportes. Por ejemplo, la armadura WARREN de la figura 6.6(a) tiene barras de 2 metros de longitud y soporta cargas en B y D. En la figura 6.6(b) dibujamos su diagrama de cuerpo libre. De las ecuaciones de equilibrio.

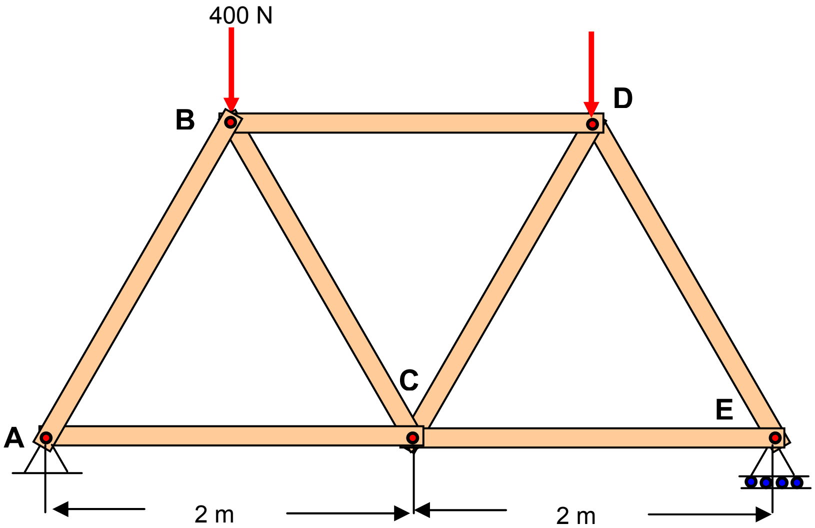


Fig. 6. 6(a) Armadura WARREN soportando dos cargas

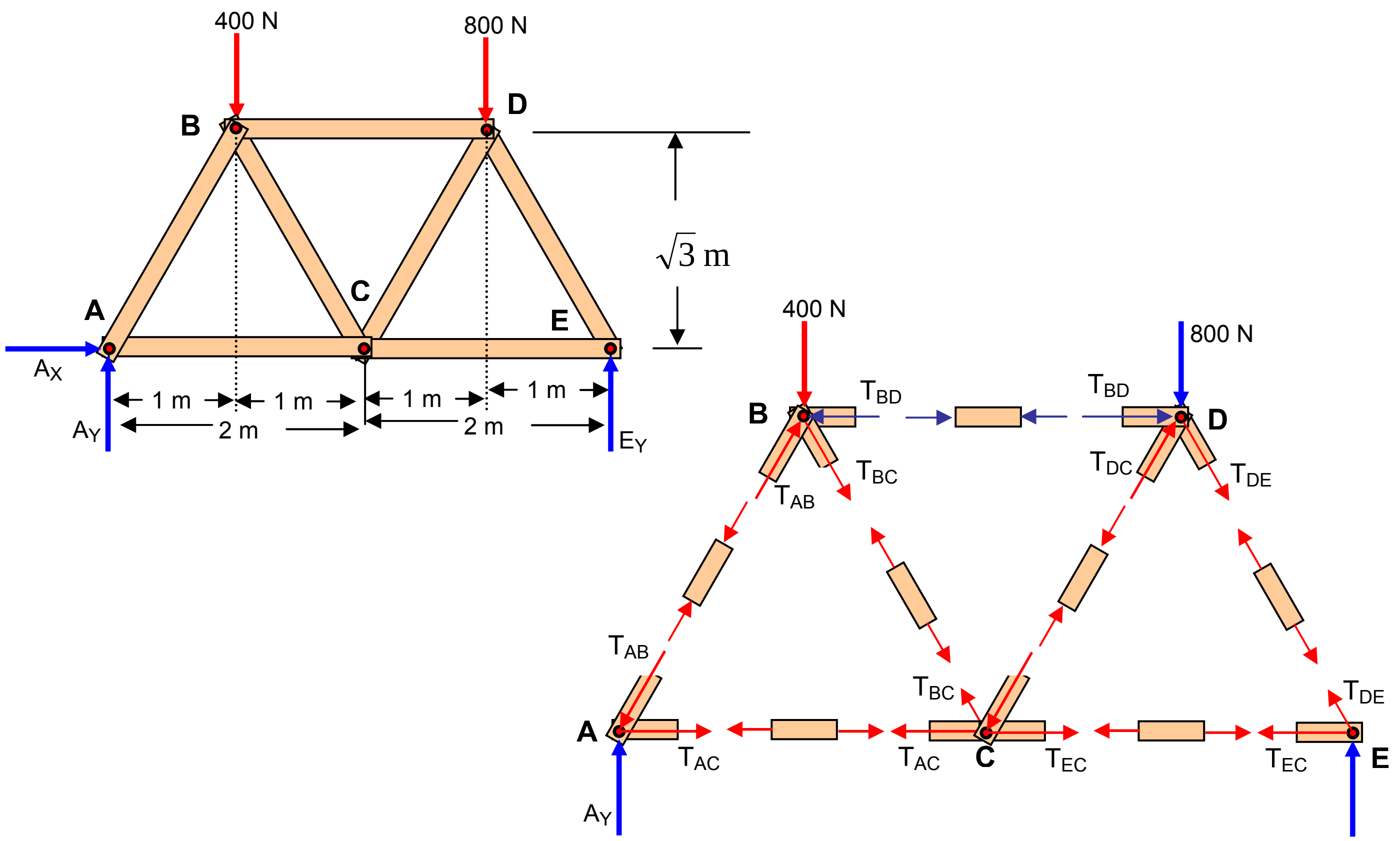


Fig. 6. 6(b) Diagrama de cuerpo libre de la armadura

$\Sigma M_A = 0$

$- 400 (1) - 800 (1 +1+1) + E_Y (1+1+1+1) = 0$

$- 400 - 800 (3) + E_Y (4) = 0$

$- 400 - 2400 + 4 E_Y = 0$

$- 2800 + 4 E_Y = 0$

$4 E_Y = 2800$

$$E_Y = \frac{2800}{4} = 700 \text{ N}$$

$E_Y = 700$ N

$\sum F_X = 0 \quad A_X = 0$

$\sum F_Y = 0$

$A_Y + E_Y - 400 - 800 = 0$

$\Sigma M_E = 0$

$- A_Y (1+1+1+1) + 400 (1+1+1) + 800 (1) = 0$

$- A_Y (4) + 400 (3) + 800 = 0$

$- 4 A_Y + 1200 + 800 = 0$

$4 A_Y = 2000$

$$A_Y = \frac{2000}{4} = 500 \text{ N}$$

$A_Y = 500$ N

El siguiente paso es elegir una junta y dibujar su diagrama de cuerpo libre. En la figura 6.7(a) aislamos la junta A cortando las barras AB y AC. Los términos T_{AB} y T_{AC} son las fuerzas axiales en las barras AB y AC respectivamente. Aunque las direcciones de las flechas que representan las fuerzas axiales desconocidas se pueden escoger arbitrariamente, observe que las hemos elegido de manera que una barra estará a tensión, si obtenemos un valor positivo para la fuerza axial. Pensamos que escoger consistentemente las direcciones de esta manera ayudara a evitar errores.

NUDO A

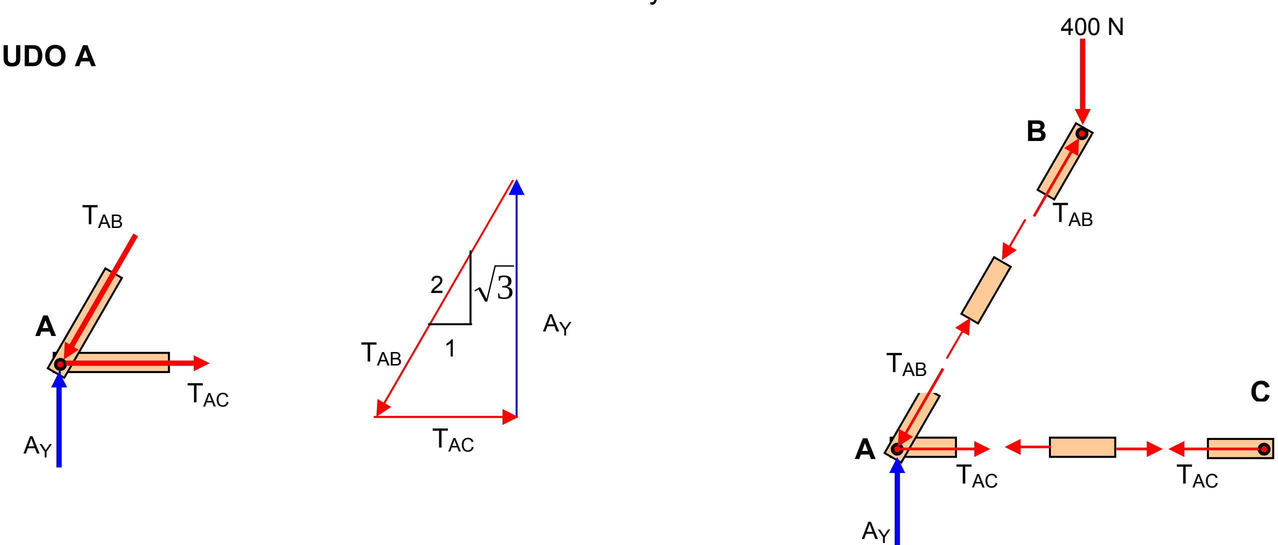


Figura 6.7(a) Obtención del diagrama de cuerpo libre de la junta A.

Las ecuaciones de equilibrio para la junta A son:

$$\frac{T_{AB}}{2} = \frac{T_{AC}}{1} = \frac{A_Y}{\sqrt{3}}$$

Hallar T_{AB}

$$\frac{T_{AB}}{2} = \frac{A_Y}{\sqrt{3}}$$

A_Y = 500 N

$$\frac{T_{AB}}{2} = \frac{500}{\sqrt{3}} = 288{,}67$$

$$T_{AB} = 2(288{,}67) = 577{,}35 \text{ N}$$

T_{AB} = 577,35 Newton(compresión)

Hallar T_{AC}

$$\frac{T_{AB}}{2} = \frac{T_{AC}}{1}$$

$$T_{AC} = \frac{T_{AB}}{2}$$

T_{AB} = 577,35 Newton

$$T_{AC} = \frac{577{,}35}{2} = 288{,}67 \text{ N}$$

T_{AC} = 288,67 Newton (Tension)

NUDO B

Luego obtenemos un diagrama de la junta B cortando las barras AB, BC y BD (Fig. 6.8 a). De las ecuaciones de equilibrio para la junta B.

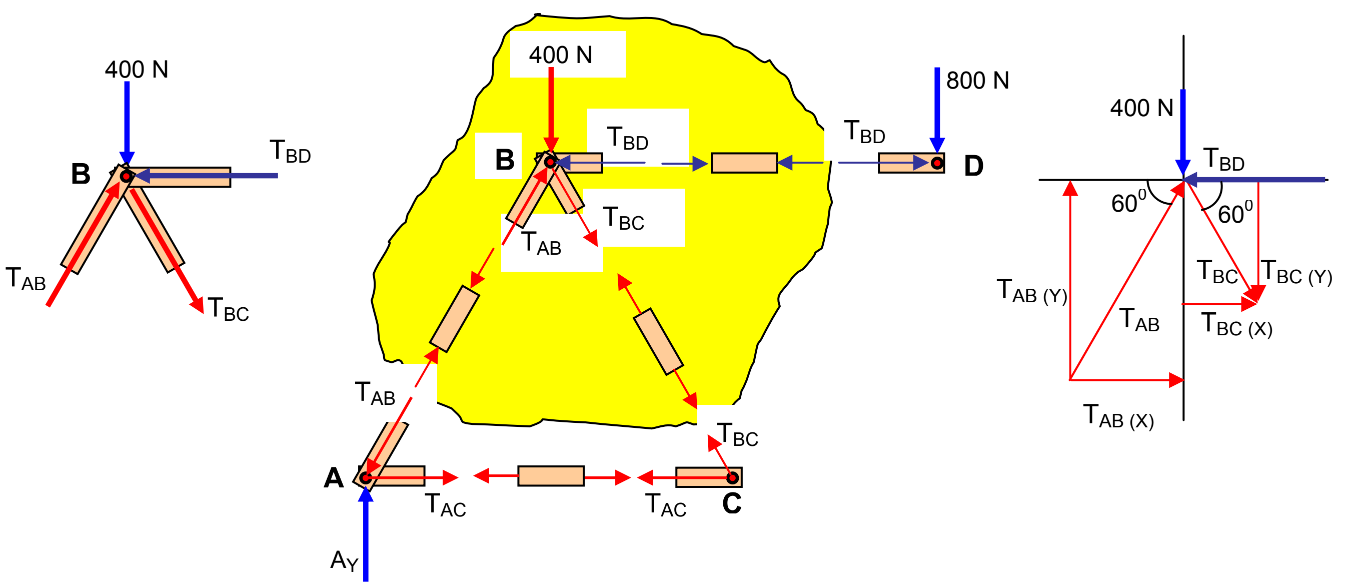


Figura 6.8(a) Obtención del diagrama de cuerpo libre de la junta B.

$$\text{sen } 60 = \frac{T_{AB(Y)}}{T_{AB}}$$

$T_{AB\,(Y)}$ = T_{AB} sen 60

$$T_{AB(Y)} = T_{AB}\left(\frac{\sqrt{3}}{2}\right)$$

Para abreviar los cálculos

$$\text{sen } 60 = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{1}{2}$$

$$T_{AB(Y)} = \left(\frac{\sqrt{3}}{2}\right) T_{AB}$$

T_{AB} = 577,35 Newton

$$T_{AB(Y)} = \left(\frac{\sqrt{3}}{2}\right)(577{,}35) = 500 \text{ N}$$

$T_{AB\ (Y)}$ = 500 N

$$\text{sen } 60 = \frac{T_{BC(Y)}}{T_{BC}}$$

$T_{BC\ (Y)}$ = T_{BC} sen 60

$$T_{BC(Y)} = T_{BC}\left(\frac{\sqrt{3}}{2}\right)$$

$$T_{BC(Y)} = \left(\frac{\sqrt{3}}{2}\right) T_{BC}$$

$$\cos 60 = \frac{T_{BC(X)}}{T_{BC}}$$

$T_{BC\ (X)}$ = T_{BC} cos 60

$$T_{BC(X)} = T_{BC}\left(\frac{1}{2}\right)$$

$$T_{BC(X)} = \left(\frac{1}{2}\right) T_{BC}$$

$$\cos 60 = \frac{T_{AB(X)}}{T_{AB}}$$

$T_{AB\ (X)}$ = T_{AB} cos 60

$$T_{AB(X)} = T_{AB}\left(\frac{1}{2}\right)$$

$$T_{AB(X)} = \left(\frac{1}{2}\right) T_{AB}$$

T_{AB} = 577,35 Newton

$$T_{AB(X)} = \frac{1}{2}(577{,}35) = 288{,}67 \text{ N}$$

$T_{AB\ (X)}$ = 288,67 N

$\sum F_Y = 0$

- 400 + $T_{AB\ (Y)}$ - $T_{BC\ (Y)}$ = 0

$T_{AB\ (Y)}$ = 500 N

- 400 + 500 - $T_{BC\ (Y)}$ = 0

100 - $T_{BC\ (Y)}$ = 0

100 = $T_{BC\ (Y)}$

$$T_{BC(Y)} = \left(\frac{\sqrt{3}}{2}\right) T_{BC}$$

100 = $T_{BC\ (Y)}$

$$100 = \left(\frac{\sqrt{3}}{2}\right) T_{BC}$$

$$T_{BC} = \left(\frac{2}{\sqrt{3}}\right) 100 = \frac{200}{\sqrt{3}} = 115{,}47 \text{ N}$$

T_{BC} = 115,47 N (compresión)

$\sum F_X = 0$

- T_{BD} + $T_{AB\ (X)}$ + $T_{BC\ (X)}$ = 0

$T_{AB\ (X)}$ = 288,67 N

$T_{BC\ (X)}$ = 57,73 Newton

- T_{BD} + 288,67 + 57,73 = 0

- T_{BD} + 346,4 = 0

T_{BD} = 346,4 Newton (compresión)

Se halla $T_{BC\ (X)}$

$$T_{BC(X)} = \left(\frac{1}{2}\right) T_{BC}$$

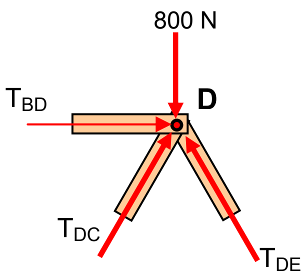
T_{BC} = 115,47 N

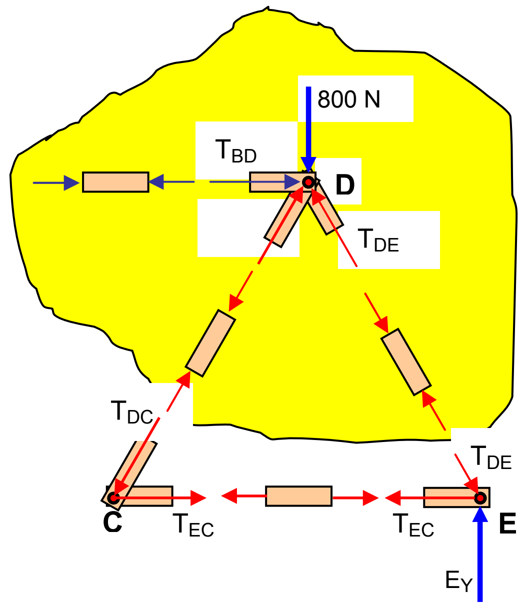
$$T_{BC(X)} = \left(\frac{1}{2}\right)(115{,}47) = 57{,}73 \text{ N}$$

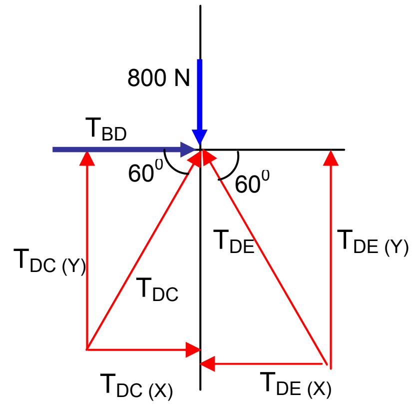
$T_{BC\ (X)}$ = 57,73 Newton

NUDO D

Luego obtenemos un diagrama de la junta D cortando las barras BD, DC y DE . De las ecuaciones de equilibrio para la junta D.







Para abreviar los cálculos

$$\text{sen } 60 = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{1}{2}$$

$$\text{sen } 60 = \frac{T_{DC(Y)}}{T_{DC}}$$

$T_{DC\ (Y)} = T_{DC}$ sen 60

$$T_{DC(Y)} = T_{DC}\left(\frac{\sqrt{3}}{2}\right)$$

$$T_{DC(Y)} = \left(\frac{\sqrt{3}}{2}\right) T_{DC}$$

$$\cos 60 = \frac{T_{DC(X)}}{T_{DC}}$$

$T_{DC\ (X)} = T_{DC}$ cos 60

$$T_{DC(X)} = T_{DC}\left(\frac{1}{2}\right)$$

$$T_{DC(Y)} = \left(\frac{\sqrt{3}}{2}\right) T_{DC}$$

$$\cos 60 = \frac{T_{DE(X)}}{T_{DE}}$$

$T_{DE\ (X)} = T_{DE}$ cos 60

$$T_{DE(X)} = T_{DE}\left(\frac{1}{2}\right)$$

$$T_{DE(X)} = \left(\frac{1}{2}\right) T_{DE}$$

$$\text{sen } 60 = \frac{T_{DE(Y)}}{T_{DE}}$$

$T_{DE\ (Y)} = T_{DE}$ sen 60

$$T_{DE(Y)} = T_{DE}\left(\frac{\sqrt{3}}{2}\right)$$

$$T_{DE(Y)} = \left(\frac{\sqrt{3}}{2}\right) T_{DE}$$

$\sum F_X = 0$

$\mathbf{T_{BD} - T_{DE\ (X)} + T_{DC\ (X)} = 0}$

T_{BD} = 346,4 Newton (compresión)

$346{,}4 \;-\; T_{DE\,(X)} + T_{DC\,(X)} = 0$

$T_{DE\,(X)} - T_{DC\,(X)} = 346{,}4$ ecuación 1

Pero:

$$T_{DE}(X) = \left(\frac{1}{2}\right) T_{DE}$$

$$T_{DC}(X) = T_{DC}\left(\frac{1}{2}\right)$$

Reemplazando en la ecuación 1

$$\left(\frac{1}{2}\right) T_{DE} - \left(\frac{1}{2}\right) T_{DC} = 346{,}4$$ **ecuación 3**

$\sum F_Y = 0$

$-800 + T_{DE\,(Y)} + T_{DC\,(Y)} = 0$

$T_{DE\,(Y)} + T_{DC\,(Y)} = 800$ ecuación 2

Pero:

$$T_{DE}(Y) = \left(\frac{\sqrt{3}}{2}\right) T_{DE}$$

$$T_{DC}(Y) = \left(\frac{\sqrt{3}}{2}\right) T_{DC}$$

Reemplazando en la ecuación 2

$$\left(\frac{\sqrt{3}}{2}\right) T_{DE} + \left(\frac{\sqrt{3}}{2}\right) T_{DC} = 800$$ **ecuación 4**

resolver ecuación 3 y ecuación 4

$$\left(\frac{1}{2}\right) T_{DE} - \left(\frac{1}{2}\right) T_{DC} = 346{,}4 \text{ multiplicar por } \left[\sqrt{3}\right]$$

$$\left(\frac{\sqrt{3}}{2}\right) T_{DE} + \left(\frac{\sqrt{3}}{2}\right) T_{DC} = 800$$

$$\left(\frac{\sqrt{3}}{2}\right) T_{DE} - \cancel{\left(\frac{\sqrt{3}}{2}\right) T_{DC}} = 346{,}4 \left[\sqrt{3}\right] = 600$$

$$\left(\frac{\sqrt{3}}{2}\right) T_{DE} + \cancel{\left(\frac{\sqrt{3}}{2}\right) T_{DC}} = 800$$

$$\left(\frac{\sqrt{3}}{2}\right)T_{DE} + \left(\frac{\sqrt{3}}{2}\right)T_{DE} = 600 + 800 = 1400$$

$$2\left(\frac{\sqrt{3}}{2}\right)T_{DE} = 1400$$

$$\sqrt{3}\,T_{DE} = 1400$$

$$T_{DE} = \frac{1400}{\sqrt{3}} = 808{,}29\text{ N}$$

T_{DE} = 808,29 Newton (compresión)

Reemplazando en la ecuación 4, se halla **T_{DC}**

$$\left(\frac{\sqrt{3}}{2}\right)T_{DE} + \left(\frac{\sqrt{3}}{2}\right)T_{DC} = 800 \quad \textbf{ecuación 4}$$

$$\left(\frac{\sqrt{3}}{2}\right)(808{,}29) + \left(\frac{\sqrt{3}}{2}\right)T_{DC} = 800$$

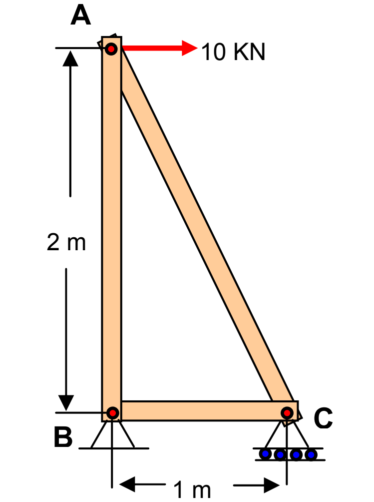
$$700 + \left(\frac{\sqrt{3}}{2}\right)T_{DC} = 800$$

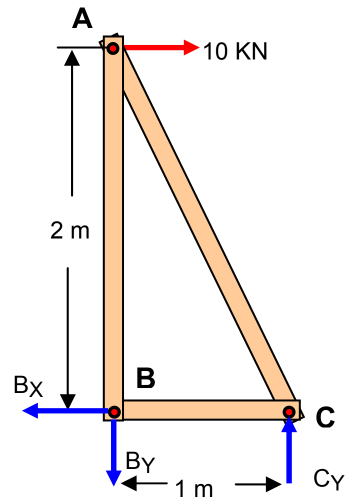
$$\left(\frac{\sqrt{3}}{2}\right)T_{DC} = 800 - 700 = 100$$

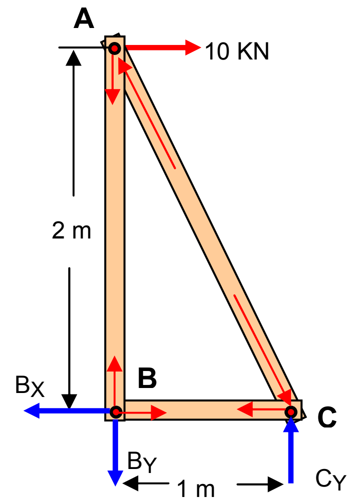
$$T_{DC} = 100\left(\frac{2}{\sqrt{3}}\right) = \frac{200}{\sqrt{3}} = 115{,}47\text{ N}$$

T_{DC} = 115,47 Newton (Tensión)

Problema 6.1 ESTATICA BEDFORD edic 4
Determine the axial forces in the members of the truss and indicate whether they are in tension (T) or compression (C)







$\Sigma M_C = 0$

$+ \quad B_Y (1) – 10 (2) = 0$

$B_Y (1) = 10 (2)$

B_Y = 20 KN

$\sum F_X = 0$

$10 – B_X = 0$

B_X = 10 KN

$\sum F_Y = 0$

$C_Y – B_Y = 0$

$C_Y = B_Y$ Pero: $B_Y = 20$ KN

C_Y = 20 KN

NUDO B

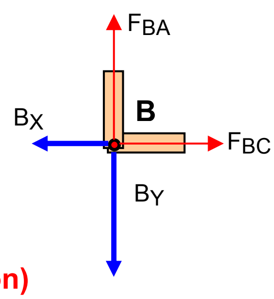
$\sum F_X = 0$

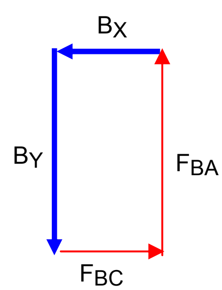
$F_{BC} – B_X = 0$

$F_{BC} = B_X$

pero: $B_X = 10$ KN

F_{BC} = 10 KN (tensión)





$\sum F_Y = 0$

$F_{BA} – B_Y = 0$
$F_{BA} = B_Y$

pero: $B_Y = 20$ KN

F_{BA} = 20 KN (tensión)

NUDO A

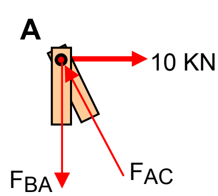
$$\frac{F_{BA}}{2} = \frac{10}{1} = \frac{F_{AC}}{\sqrt{5}}$$

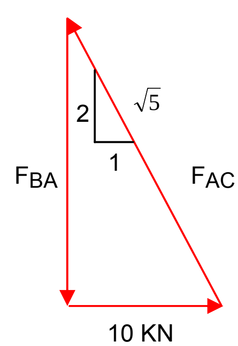
Hallamos F_{AC}

$$\frac{10}{1} = \frac{F_{AC}}{\sqrt{5}}$$

$$F_{AC} = 10\left(\sqrt{5}\right) = 22{,}36 KN$$

F_{AC} = 22,36 KN (compresión)



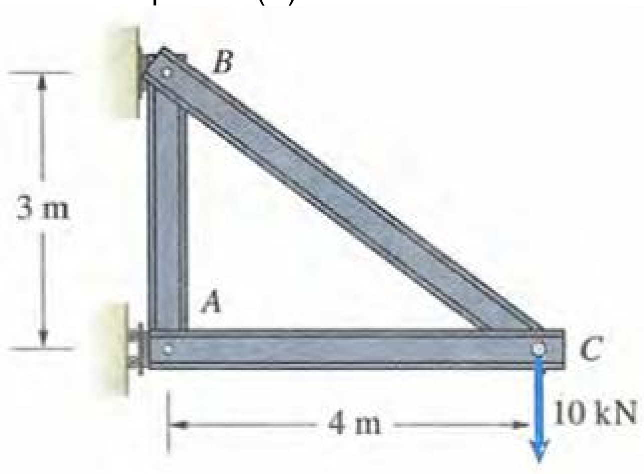


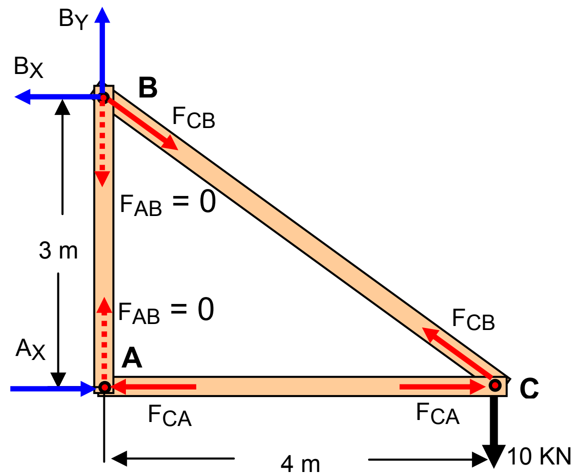
Problema 6.4 ESTATICA BEDFORD edic 4

La armadura mostrada soporta una carga de 10 kN en C.

a) Dibuje el diagrama de cuerpo libre de toda la armadura y determine las reacciones en sus soportes

b) Determine las fuerzas axiales en las barras. Indique si se encuentran a tensión (T) o a compresión (C) .





$\Sigma M_B = 0$

+ $A_X (3) - 10 (4) = 0$

$A_X (3) = 10 (4)$

$3 A_X = 40$

$$A_X = \frac{40}{3} = 13{,}33 KN$$

$A_X = 13{,}33$ KN

$\sum F_Y = 0$
$B_Y - 10 = 0$
$B_Y = 10$ KN

$\Sigma M_A = 0$

+ $B_X (3) - 10 (4) = 0$

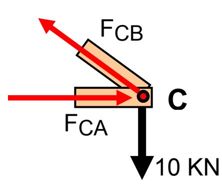
$B_X (3) = 10 (4)$

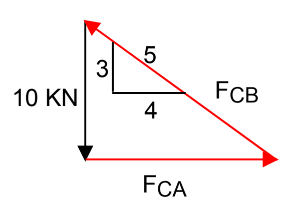
$3 B_X = 40$

$$B_X = \frac{40}{3} = 13{,}33 KN$$

$B_X = 13{,}33$ KN

NUDO C





$$\frac{F_{CB}}{5} = \frac{F_{CA}}{4} = \frac{10}{3}$$

Hallar F_{CB}

$$\frac{F_{CB}}{5} = \frac{10}{3}$$

$$F_{CB} = \frac{(5)10}{3} = 16{,}66 \text{ KN}$$

F_{CB} = 16,66 kN (Tensión)

Hallar F_{CA}

$$\frac{F_{CA}}{4} = \frac{10}{3}$$

$$F_{CA} = \frac{(4)10}{3} = 13{,}33 \text{ KN}$$

F_{CA} = 13,33 kN (compresión)

NUDO A

$\sum F_Y = 0$ **$F_{AB} = 0$**

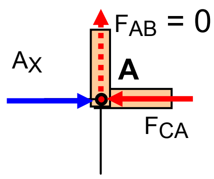
$\sum F_X = 0$

$A_X - F_{CA} = 0$

$A_X = F_{CA}$

Pero: F_{CA} = 13,33 kN

$A_X = F_{CA}$ =13,33 kN



A_X = 13,33 KN

B_Y = 10 KN

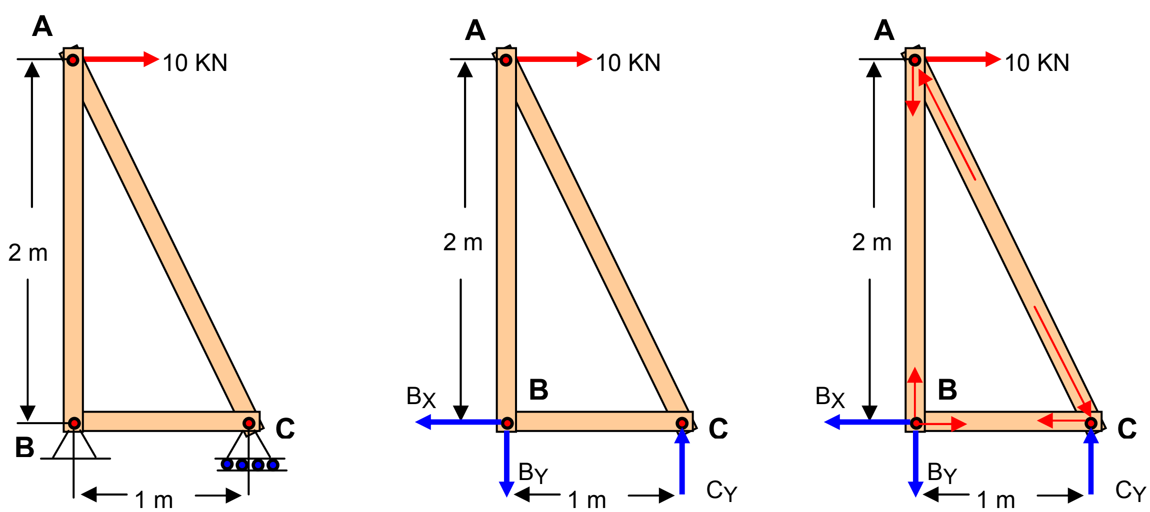
B_X = 13,33 KN

F_{CB} = 16,66 kN (Tensión)

F_{CA} = 13,33 kN (compresión)

F_{AB} = 0

Problema 6.1 ESTATICA BEDFORD edic 4
Determine the axial forces in the members of the truss and indicate whether they are in tension (T) or compression (C)



$\sum M_C = 0$

$B_Y (1) - 10 (2) = 0$

$B_Y (1) = 10 (2)$

$B_Y = 20$ KN

$\sum F_X = 0$

$10 - B_X = 0$

$B_X = 10$ KN

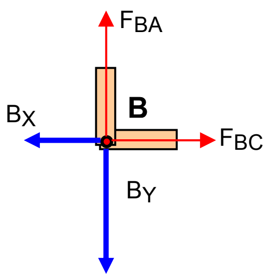
$\sum F_Y = 0$

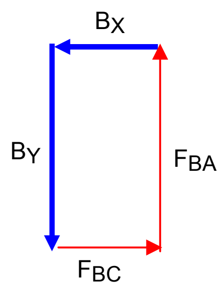
$C_Y - B_Y = 0$

$C_Y = B_Y$ Pero: $B_Y = 20$ KN

$C_Y = 20$ KN

NUDO B





$\sum F_X = 0$

$F_{BC} - B_X = 0$

$F_{BC} = B_X$

pero: $B_X = 10$ KN

$F_{BC} = 10$ KN (tensión)

$\sum F_Y = 0$

$F_{BA} - B_Y = 0$

$F_{BA} = B_Y$

pero: $B_Y = 20$ KN

$F_{BA} = 20$ KN (tensión)

NUDO A

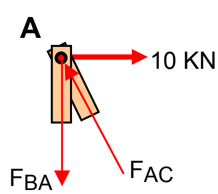
$$\frac{F_{BA}}{2} = \frac{10}{1} = \frac{F_{AC}}{\sqrt{5}}$$

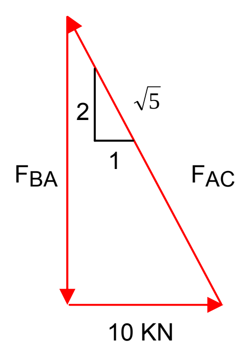
Hallamos F_{AC}

$$\frac{10}{1} = \frac{F_{AC}}{\sqrt{5}}$$

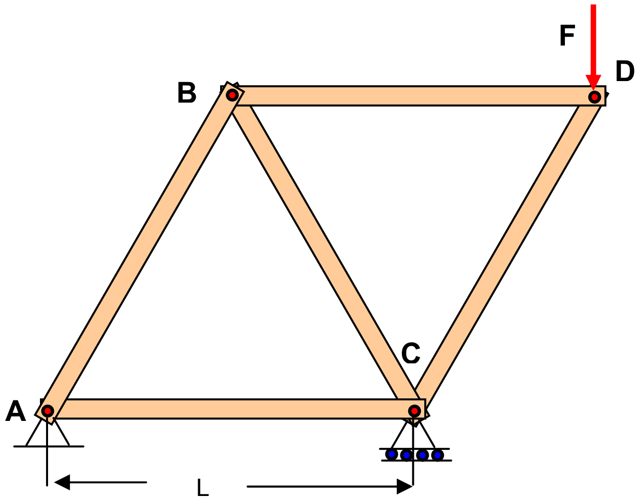
$$F_{AC} = 10\left(\sqrt{5}\right) = 22{,}36\,KN$$

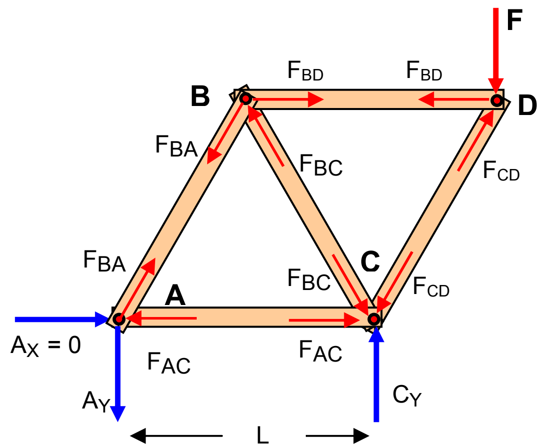
$F_{AC} = 22,36$ KN (compresión)





Problema 6.4 ESTATICA BEDFORD edic 4
The members of the truss are all of lenght L. Determine the axial forces in the members and indicate whether they are in tension (T) or compression (C)





NUDO D

$\Sigma M_C = 0$

$+ \quad A_Y (L) - F (L/2) = 0$

$A_Y (\cancel{L}) = F (\cancel{L}/2)$

$A_Y = ½ F$

$\Sigma M_A = 0$

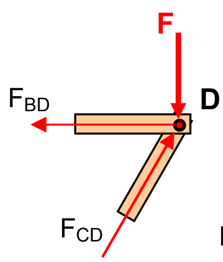
$+ \quad C_Y (L) - F (L + L/2) = 0$

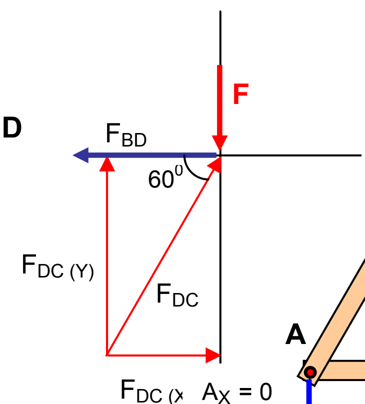
$C_Y (L) - F (3/2 L) = 0$

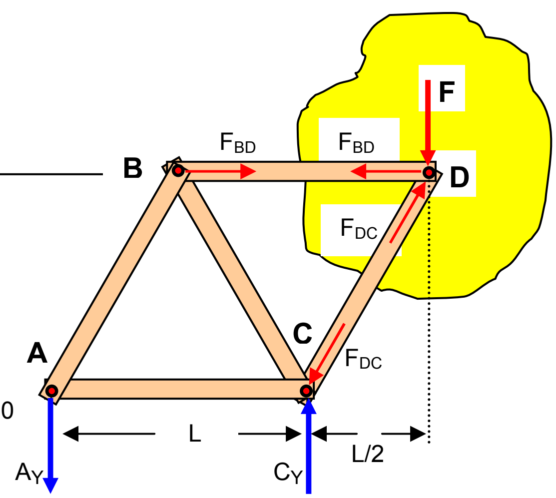
$C_Y (\cancel{L}) = F (3/2 \cancel{L})$

$C_Y = F (3/2)$

$C_Y = 3/2 F$







Para abreviar los cálculos

$$\text{sen } 60 = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{1}{2}$$

$$\text{sen } 60 = \frac{F_{DC(Y)}}{F_{DC}}$$

$F_{DC (Y)} = F_{DC} \text{ sen } 60$

$$F_{DC(Y)} = F_{DC}\left(\frac{\sqrt{3}}{2}\right)$$

$$F_{DC(Y)} = \left(\frac{\sqrt{3}}{2}\right) F_{DC}$$

$\sum F_Y = 0$

$- F + F_{DC\,(Y)} = 0$

$F = F_{DC\,(Y)}$

Pero:

$F_{DC\,(Y)} = F_{DC}$ sen 60
$F = F_{DC}$ sen 60

DESPEJANDO F_{DC}

$$F_{DC} = \frac{1}{\text{sen } 60}(F) = 1{,}154\ F$$

F_{DC} = 1,154 F (Compresion)

$\sum F_X = 0$

$- F_{BD} + F_{DC\,(X)} = 0$

$F_{BD} = F_{DC\,(X)}$

Pero:
$F_{DC\,(X)} = F_{DC}$ cos 60

F_{BD} = F_{DC} cos 60

Pero:
$F_{DC} = 1{,}154$ F

F_{BD} = (1,154 F) cos 60

F_{BD} = 0,577 F (tensión)

$$\cos 60 = \frac{F_{DC(X)}}{F_{DC}}$$

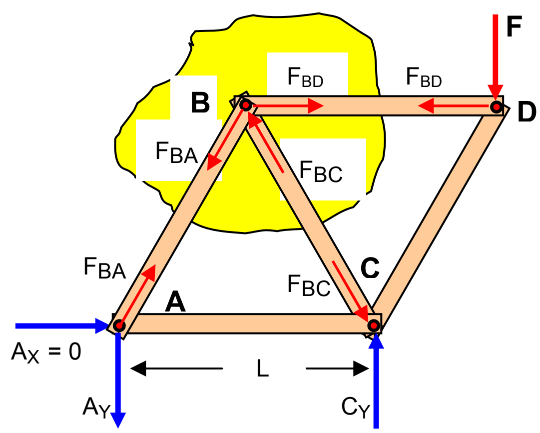
$F_{DC\,(X)} = F_{DC}$ cos 60

$$F_{DC(X)} = F_{DC}\left(\frac{1}{2}\right)$$

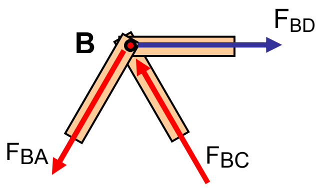
$\sum F_X = 0 \quad A_X = 0$

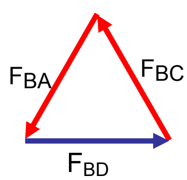
$\sum F_Y = 0$

$A_Y + E_Y – 400 - 800 = 0$



NUDO B





$$\text{sen } 60 = \frac{F_{BA(Y)}}{T_{AB}}$$

$F_{BA\,(Y)} = T_{BA}$ sen 60

$$F_{BA(Y)} = F_{BA}\left(\frac{\sqrt{3}}{2}\right)$$

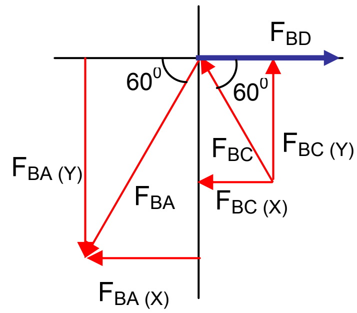
$$F_{BA(Y)} = \left(\frac{\sqrt{3}}{2}\right) F_{BA}$$

$$\cos 60 = \frac{F_{BA(X)}}{F_{BA}}$$

$F_{BA\ (X)} = F_{BA} \cos 60$

$$F_{BA(X)} = F_{BA}\left(\frac{1}{2}\right)$$

$$F_{BA(X)} = \left(\frac{1}{2}\right) F_{BA}$$



$$\text{sen } 60 = \frac{F_{BC(Y)}}{F_{BC}}$$

$F_{BC\ (Y)} = T_{BC}$ sen 60

$$F_{BC(Y)} = F_{BC}\left(\frac{\sqrt{3}}{2}\right)$$

$$F_{BC(Y)} = \left(\frac{\sqrt{3}}{2}\right) F_{BC}$$

$$\cos 60 = \frac{F_{BC(x)}}{F_{BC}}$$

$F_{BC\ (X)} = F_{BC} \cos 60$

$$F_{BC\ (X)} = F_{BC}\left(\frac{1}{2}\right)$$

$\sum F_X = 0$

$F_{BD} - F_{BC\ (X)} - F_{BA\ (X)} = 0$

$$F_{BD} - F_{BC(X)} - F_{BA\ (X)} = 0$$

$$F_{BC(X)} + F_{BA\ (X)} = F_{BD}$$

Para abreviar los cálculos

$$\text{sen } 60 = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{1}{2}$$

PERO:

F_{BD} = 0,577 F

$$F_{BC(X)} + F_{BA\ (X)} = 0{,}577\ F$$

$$\left(\frac{1}{2}\right) F_{BC} + \left(\frac{1}{2}\right) F_{BA} = 0{,}577\ F$$ **(ECUACIÓN 1)**

$\sum F_Y = 0$

$F_{BC\ (Y)} - F_{BA\ (Y)} = 0$

$$\left(\frac{\sqrt{3}}{2}\right) F_{BC} - \left(\frac{\sqrt{3}}{2}\right) F_{BA} = 0$$ **(ECUACIÓN 2)**

resolver ecuación 1 y ecuación 2

$$\left(\frac{1}{2}\right) F_{BC} + \left(\frac{1}{2}\right) F_{BA} = 0{,}577\ F \text{ multiplicar por } \left[\sqrt{3}\right]$$

$$\left(\frac{\sqrt{3}}{2}\right) F_{BC} - \left(\frac{\sqrt{3}}{2}\right) F_{BA} = 0$$

$$\left(\frac{\sqrt{3}}{2}\right) F_{BC} + \cancel{\left(\frac{\sqrt{3}}{2}\right) F_{BA}} = \left(\sqrt{3}\right)(0{,}577\ F)$$

$$\left(\frac{\sqrt{3}}{2}\right) F_{BC} - \cancel{\left(\frac{\sqrt{3}}{2}\right) F_{BA}} = 0$$

$$2\left(\frac{\sqrt{3}}{2}\right) F_{BC} = F$$

$$\sqrt{3}\, F_{BC} = F$$

$$F_{BC} = \left(\frac{1}{\sqrt{3}}\right) F$$

F_{BC} = 0,577 F (compresión)

Reemplazando en la ecuación 2

$$\left(\frac{\sqrt{3}}{2}\right) F_{BC} - \left(\frac{\sqrt{3}}{2}\right) F_{BA} = 0 \quad \textbf{(ECUACIÓN 2)}$$

$$\left(\frac{\sqrt{3}}{2}\right)(0{,}577\ F) - \left(\frac{\sqrt{3}}{2}\right) F_{BA} = 0$$

$$\cancel{\left(\frac{\sqrt{3}}{2}\right)}(0{,}577\ F) = \cancel{\left(\frac{\sqrt{3}}{2}\right)} F_{BA}$$

Cancelando terminos semejantes

$$(0{,}577\ F) = F_{BA}$$

F_{BA} = 0,577 F (tensión)

NUDO A

$$\frac{F_{BA}}{L} = \frac{F_{AC}}{L/2}$$

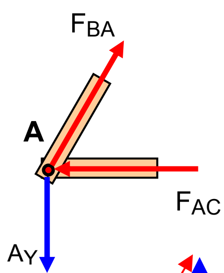
$$\frac{F_{BA}}{\cancel{L}} = \frac{2\ F_{AC}}{\cancel{L}}$$

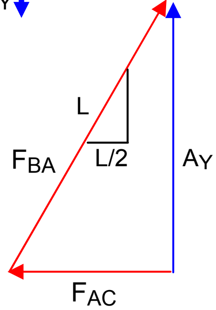
Cancelando términos semejantes

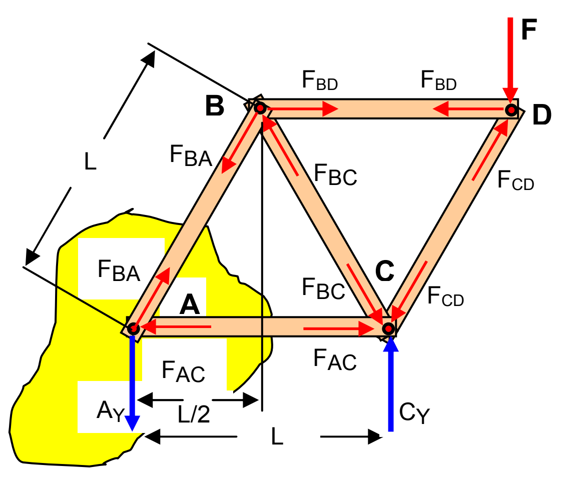
F_{BA} = 2 F_{AC}

Pero: F_{BA} = 0,577 F

0,577 F = 2 F_{AC}





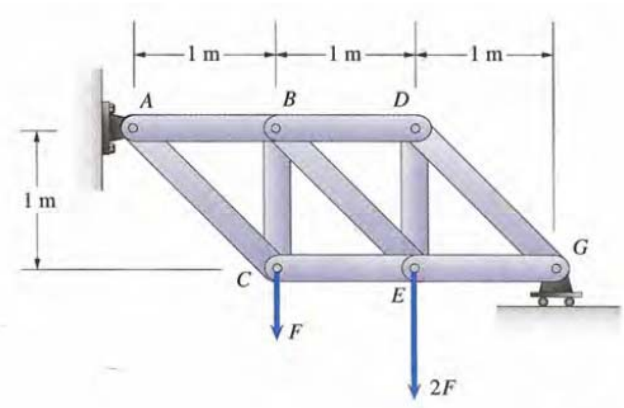


$$F_{AC} = \frac{0,577}{2} F$$

F_{AC} = 0,288 F (Compresión)

Problema 6.13 bedford edic 4

La armadura recibe cargas en C y E. Si F = 3 KN, cuales son las fuerzas axiales BC y BE?



$\Sigma M_G = 0$

+ 6 (1) + 3 (1 +1) - A_Y (1+1+1) = 0

6 (1) + 3 (2) - A_Y (3) = 0

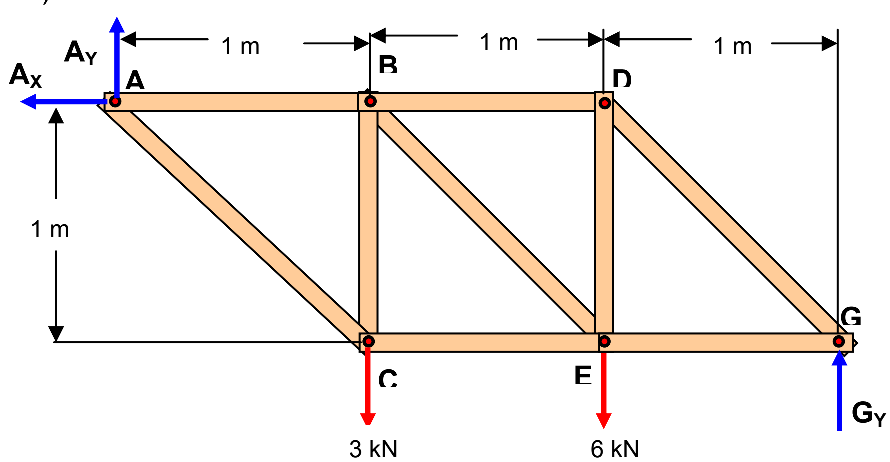
6 + 6 – 3 A_Y = 0

6 + 6 = 3 A_Y

12 = 3 A_Y

$$A_Y = \frac{12}{3} = 4\,KN$$

A_Y = 4 KN



$\Sigma M_A = 0$

+ - 3 (1) - 6 (1 +1) + G_Y (1+1+1) = 0

- 3 - 6 (2) + G_Y (3) = 0

- 3 - 12 + 3 G_Y = 0

- 15 + 3 G_Y = 0

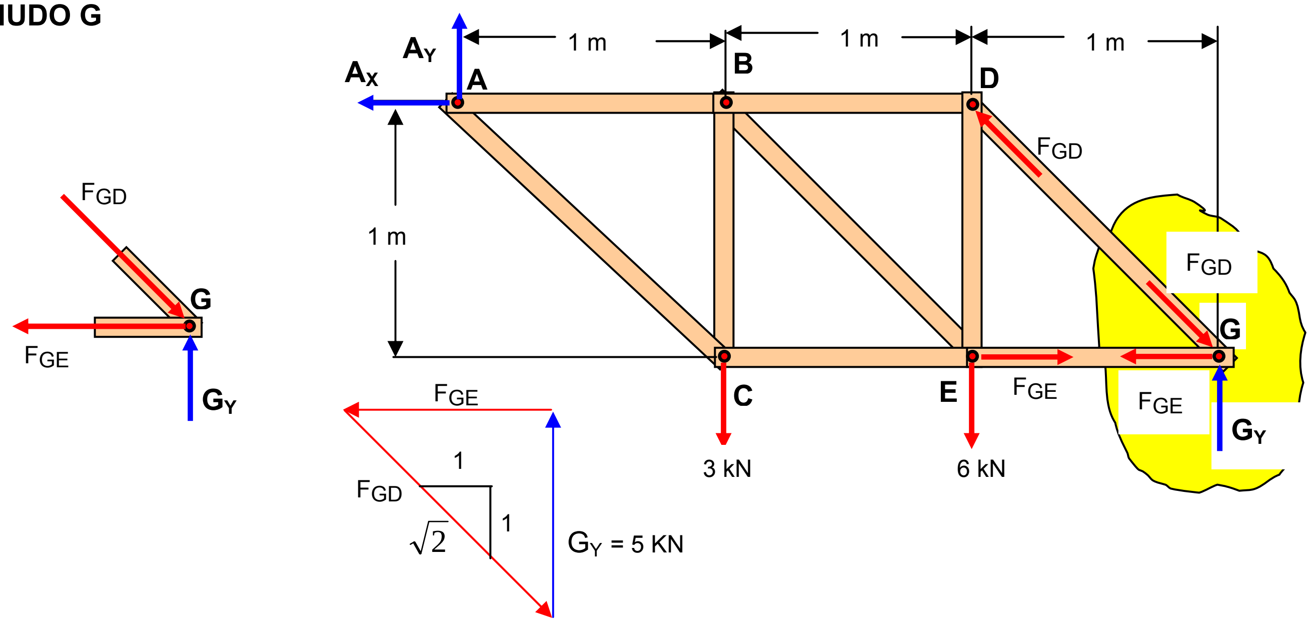
3 G_Y = 15

$$G_Y = \frac{15}{3} = 5\,KN$$

G_Y = 5 KN

$\sum F_X = 0$ $A_X = 0$

NUDO G



Las ecuaciones de equilibrio para la junta G son:

$$\frac{F_{GD}}{\sqrt{2}} = \frac{F_{GE}}{1} = \frac{5}{1}$$

Hallar F_{GD}

$$\frac{F_{GD}}{\sqrt{2}} = 5$$

$$F_{GD} = \sqrt{2}\,(5)$$

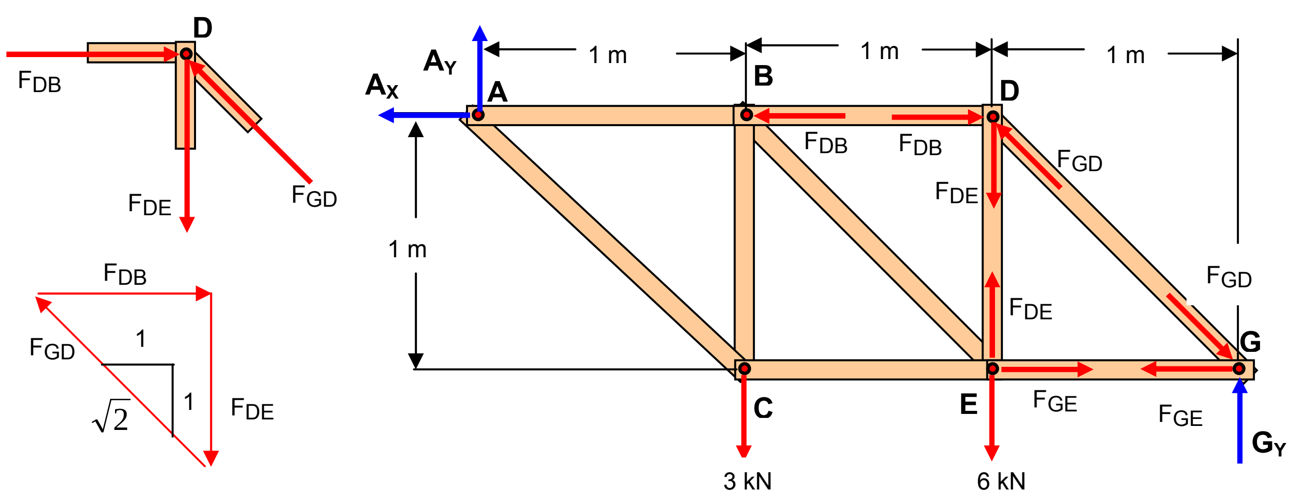
F_{GD} = 7,071 KN (compresión)

Hallar F_{GE}

$$\frac{F_{GE}}{1} = \frac{5}{1}$$

F_{GE} = 5 KN (Tensión)

NUDO D



Las ecuaciones de equilibrio para la junta D son:

$$\frac{F_{GD}}{\sqrt{2}} = \frac{F_{DE}}{1} = \frac{F_{DB}}{1}$$

PERO: **F_{GD} = 7,071 KN**

$$\frac{7{,}071}{\sqrt{2}} = \frac{F_{DE}}{1} = \frac{F_{DB}}{1}$$

$$5 = F_{DE} = F_{DB}$$

Hallar F_{DE}

$$5 = F_{DE}$$

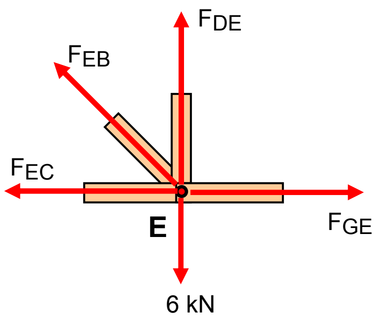
F_{DE} = 5 KN (TENSION)

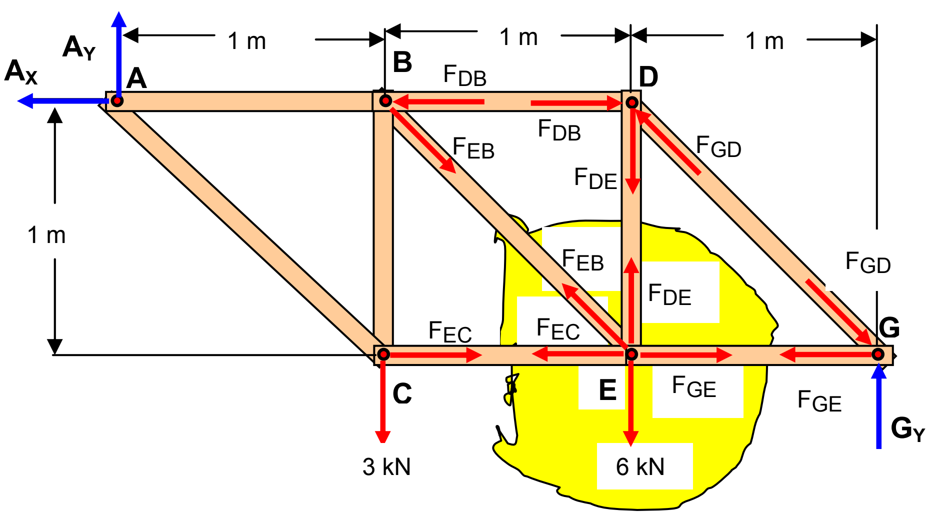
Hallar F_{DB}

$$5 = F_{DB}$$

F_{DB} = 5 KN (compresion)

NUDO E





$$\operatorname{sen} 45 = \frac{F_{EB(Y)}}{F_{EB}}$$

$F_{EB\,(Y)}$ = F_{EB} sen 45

$$F_{EB(Y)} = F_{EB}\left(\frac{\sqrt{2}}{2}\right)$$

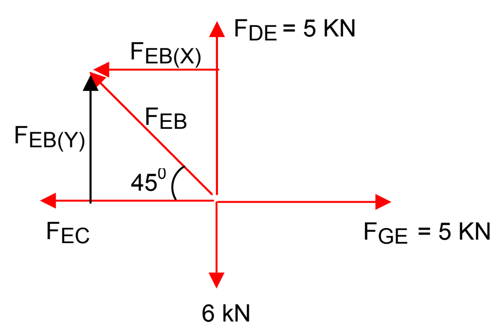
$$F_{EB(Y)} = \left(\frac{\sqrt{2}}{2}\right) F_{EB}$$

$$\cos 45 = \frac{F_{EB(X)}}{F_{EB}}$$

$F_{EB\,(X)}$ = F_{EB} cos 45

$$F_{EB(X)} = F_{EB}\left(\frac{\sqrt{2}}{2}\right)$$

$$F_{EB(X)} = \left(\frac{\sqrt{2}}{2}\right) F_{EB}$$



$\sum F_Y = 0$

$F_{DE} - 6 + F_{EB(Y)} = 0$

PERO: $F_{DE} = 5$ kN

$5 - 6 + F_{EB(Y)} = 0$

$-1 + F_{EB(Y)} = 0$

$F_{EB(Y)} = 1$ KN

$$F_{EB} = \frac{F_{EB(Y)}}{sen 45} = \frac{1}{\text{sen } 45} = 1{,}414 \text{ kN}$$

$F_{EB} = 1{,}414$ KN (tension)

$F_{EB\,(X)} = F_{EB} \cos 45$

$F_{EB\,(X)} = (1{,}414) \cos 45$

$F_{EB\,(X)} = 1$ KN

$\sum F_X = 0$

$F_{GE} - F_{EC} - F_{EB\,(X)} = 0$

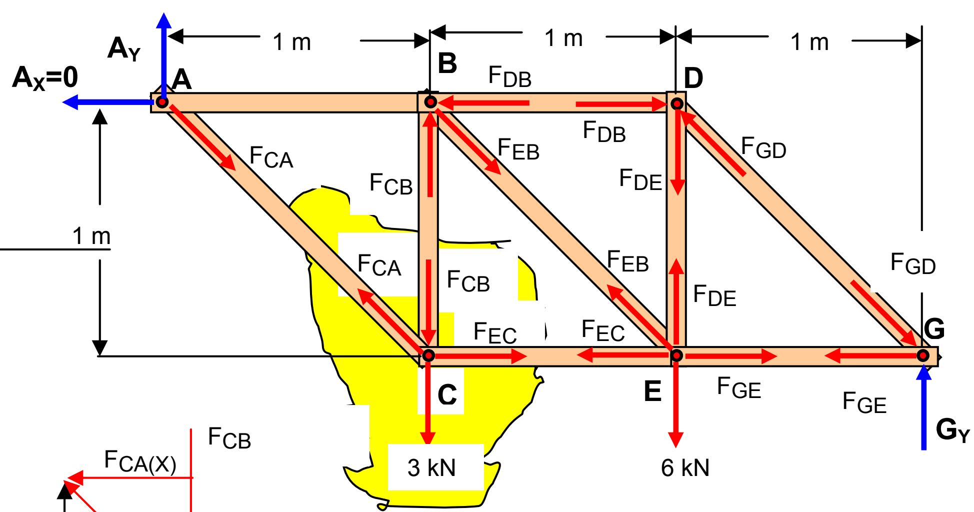
PERO:
$F_{GE} = 5$ kN
$F_{EB\,(X)} = 1$ KN

$F_{GE} - F_{EC} - F_{EB\,(X)} = 0$

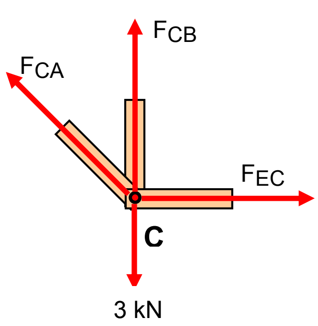
$5 - F_{EC} - 1 = 0$

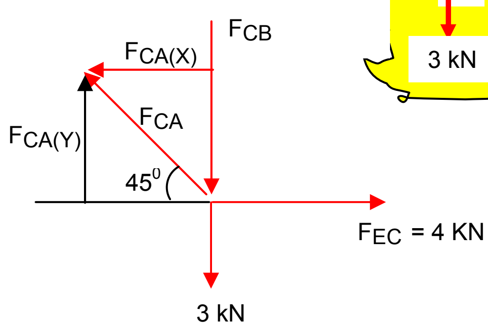
$4 - F_{EC} = 0$

$F_{EC} = 4$ KN (tension)



NUDO C





$$\text{sen } 45 = \frac{F_{CA(Y)}}{F_{CA}}$$

$F_{CA\,(Y)}$ = F_{CA} sen 45

$$F_{CA}(Y) = F_{CA}\left(\frac{\sqrt{2}}{2}\right)$$

$$F_{CA}(Y) = \left(\frac{\sqrt{2}}{2}\right)F_{CA}$$

$$\cos 45 = \frac{F_{CA}(X)}{F_{CA}}$$

$F_{CA\,(X)}$ = F_{CA} cos 45

$$F_{CA}(X) = F_{CA}\left(\frac{\sqrt{2}}{2}\right)$$

$$F_{CA}(X) = \left(\frac{\sqrt{2}}{2}\right)F_{CA}$$

$\sum F_X = 0$

F_{EC} - $F_{AC\,(X)}$ = 0

F_{EC} = $F_{AC\,(X)}$

PERO:
F_{EC} = 4 kN

$F_{AC\,(X)}$ = 4 kN

$F_{CA\,(X)}$ = F_{CA} cos 45

$$F_{CA} = \frac{F_{CA}(X)}{\cos 45} = \frac{4}{0{,}7071} = 5{,}656\text{kN}$$

F_{CA} = 5,656 KN (tension)

$$F_{CA}(Y) = \left(\frac{\sqrt{2}}{2}\right)F_{CA}$$

$$F_{CA}(Y) = \left(\frac{\sqrt{2}}{2}\right)5{,}656 = 4\text{ KN}$$

$F_{CA\,(Y)}$ = 4 kN

$\sum F_Y = 0$

- F_{CB} - 3 + $F_{CA(Y)}$ = 0

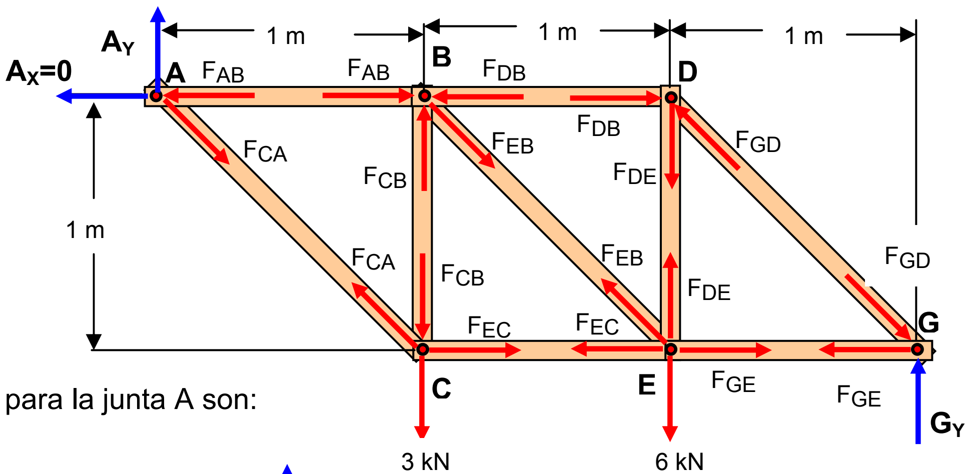
PERO:
$F_{CA\,(Y)}$ = 4 kN

- F_{CB} - 3 + 4 = 0

- F_{CB} + 1 = 0

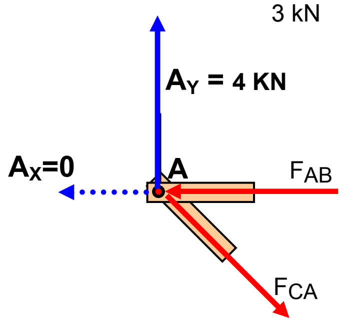
F_{CB} = 1 KN (compresión)

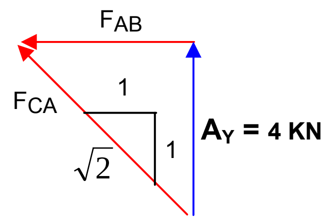
NUDO A



Las ecuaciones de equilibrio para la junta A son:

$$\frac{F_{CA}}{\sqrt{2}} = \frac{F_{AB}}{1} = \frac{A_Y}{1}$$





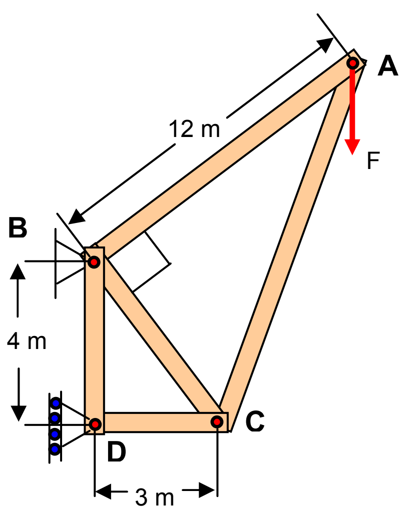
PERO: A_Y **= 4 KN**

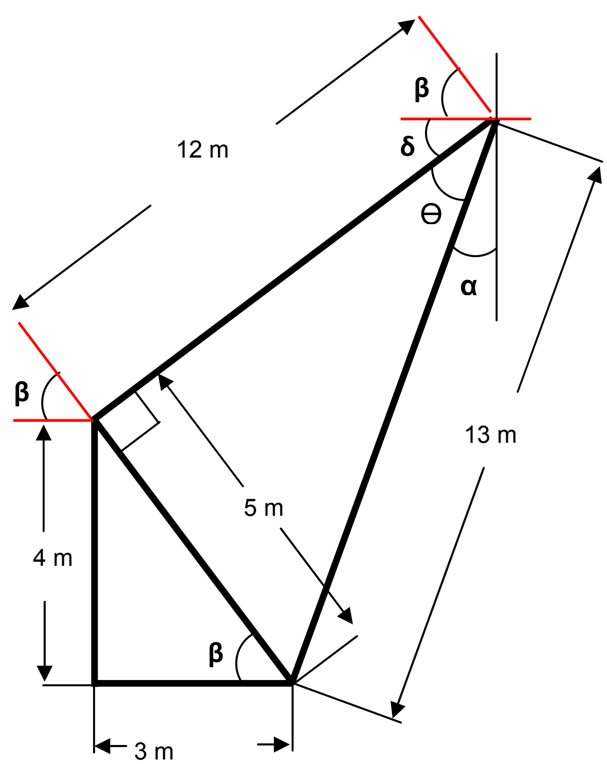
$$\frac{F_{AB}}{1} = \frac{A_Y}{1}$$

F_{AB} = 4 KN (compresión)

Problema 6.14 bedford edic 4

If you don't want the members of the truss to be subjected to an axial load (tension or compression) greater than 20 kn, what is the largest acceptable magnitude of the downward force F?





$$\text{tg}\,\theta = \frac{5}{12} = 0{,}4166$$

ϴ = arc tg (0,4166)

ϴ = 22,61^0

$$\text{tg}\,\beta = \frac{4}{3} = 1{,}3333$$

β = arc tg (1,3333)

β = 53,12⁰

β + δ = 90⁰

δ = 90⁰ - β

δ = 90⁰ - 53,12⁰

δ = 36,87⁰

δ + Θ + α = 90⁰

pero:
δ = 36,87⁰
Θ = 22,61⁰

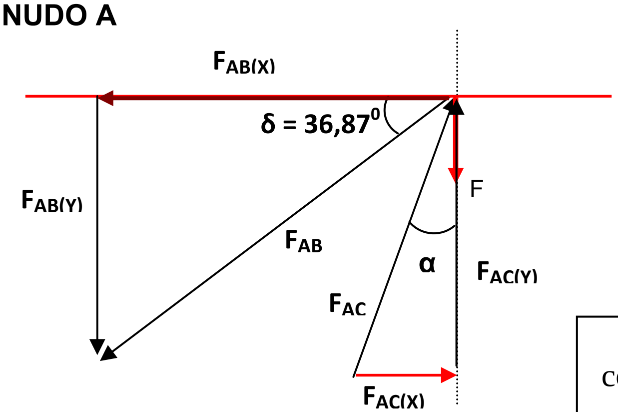
δ + Θ + α = 90⁰

36,87 + 22,61 + α = 90⁰

α = 90⁰ - 36,87 - 22,61

α = 30,52⁰

NUDO A



$$\text{sen } 36{,}87 = \frac{F_{AB(Y)}}{F_{AB}}$$

$F_{AB\,(Y)} = F_{AB}$ sen 36,87

$$F_{AB(Y)} = (0{,}6)\,F_{AB}$$

$$\cos 36{,}87 = \frac{F_{AB(X)}}{F_{AB}}$$

$F_{AB\,(X)} = F_{AB}$ cos 36,87

$$F_{AB(X)} = (0{,}8)\,F_{AB}$$

$$\text{sen } \alpha = \frac{F_{AC(X)}}{F_{AC}}$$

$$\text{sen } 30{,}52 = \frac{F_{AC(X)}}{F_{AC}}$$

$F_{AC\,(X)} = F_{AC}$ sen 30,52

$$F_{AC(X)} = (0{,}507)\,F_{AC}$$

$$\cos 30{,}52 = \frac{F_{AC(Y)}}{F_{AC}}$$

$F_{AC\,(Y)} = F_{AC}$ cos 30,52

$$F_{AC(Y)} = (0{,}8614)\,F_{AC}$$

$\sum F_X = 0$
$F_{AC(X)} - F_{AB\,(X)} = 0$

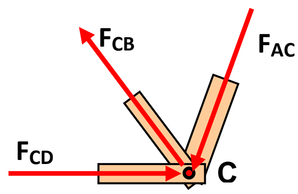
0,507 F_{AC} - 0,8 F_{AB} = 0 ECUACION 1

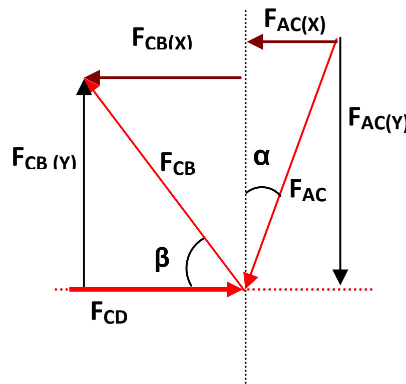
$\sum F_Y = 0$

$F_{AC\,(Y)} - F - F_{AB\,(Y)} = 0$

0,8614 F_{AC} - F - 0,6 F_{AB} = 0 ECUACION 2

NUDO C





$\beta = 53{,}12^0$

$$\text{sen}\, 53{,}12 = \frac{F_{CB(Y)}}{F_{CB}}$$

$F_{CB\,(Y)} = F_{CB}$ sen 53,12

$$F_{CB(Y)} = (0{,}7998)\, F_{CB}$$

$$\cos 53{,}12 = \frac{F_{CB(X)}}{F_{CB}}$$

$F_{CB\,(X)} = F_{CB} \cos 53{,}12$

$$F_{CB(X)} = (0{,}6)\, F_{CB}$$

$$F_{AC(X)} = (0{,}507)\, F_{AC}$$

$$F_{AC(Y)} = (0{,}8614)\, F_{AC}$$

$\sum F_X = 0$

$F_{CD} - F_{AC(X)} - F_{CB\,(X)} = 0$

$F_{CD} - 0{,}507 F_{AC} - 0{,}6\, F_{CB} = 0$ **ECUACION 3**

$\sum F_Y = 0$

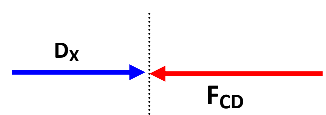
$F_{CB\,(Y)} - F_{AC\,(Y)} = 0$

$0{,}7998\, F_{CB} - 0{,}8614\, F_{AC} = 0$ **ECUACION 4**

NUDO D

$\sum F_X = 0$

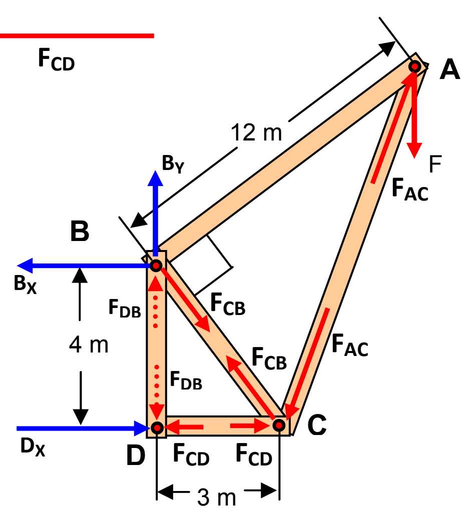
$D_X - F_{CD} = 0$ **ECUACION 5**



$0{,}507\, F_{AC} - 0{,}8\, F_{AB} = 0$ ECUACION 1
$0{,}8614\, F_{AC} - F - 0{,}6\, F_{AB} = 0$ ECUACION 2
$F_{CD} - 0{,}507 F_{AC} - 0{,}6\, F_{CB} = 0$ ECUACION 3
$0{,}7998\, F_{CB} - 0{,}8614\, F_{AC} = 0$ ECUACION 4
$D_X - F_{CD} = 0$ ECUACION 5

DESPEJAMOS F en la ecuación 2
$0{,}8614\, F_{AC} - F - 0{,}6\, F_{AB} = 0$ ECUACION 2

$0{,}8614\, F_{AC} - 0{,}6\, F_{AB} = F$ ECUACION 6



Resolver la ecuación 1
$0{,}507\, F_{AC} - 0{,}8\, F_{AB} = 0$
$0{,}507\, F_{AC} = 0{,}8\, F_{AB}$

Despejando F_{AC}

$$F_{AC} = \frac{0,8}{0,507} F_{AB} = 1,577\, F_{AB}$$

F_{AC} = 1,577 F_{AB}

Reemplazar **F_{AC}** en la ecuación 6
$0,8614\, F_{AC} - 0,6\, F_{AB} = F$ ECUACION 6

$0,8614\,(1,577\, F_{AB}) - 0,6\, F_{AB} = F$
$1,3592\, F_{AB} - 0,6\, F_{AB} = F$

$0,7592\, F_{AB} = F$

Despejando F_{AB}

$$F_{AB} = \frac{1}{0,7592} F = 1,317\, F$$

F_{AB} = 1,317 F

Reemplazar **F_{AB}** en la ecuación 6
$0,8614\, F_{AC} - 0,6\, F_{AB} = F$ ECUACION 6

$0,8614\, F_{AC} - 0,6\,(1,317\, F) = F$

$0,8614\, F_{AC} - 0,79\, F = F$

$0,8614\, F_{AC} = F + 0,79\, F$

$0,8614\, F_{AC} = 1,79\, F$

$$F_{AC} = \frac{1,79}{0,8614} F = 2,078\, F$$

F_{AC} = 2,078 F

Reemplazar F_{AC} en la ecuación 4
$0,7998\, F_{CB} - 0,8614\, F_{AC} = 0$ ECUACION 4

$0,7998\, F_{CB} - 0,8614\,(2,078\, F) = 0$
$0,7998\, F_{CB} - 1,79\, F = 0$

$0,7998\, F_{CB} = 1,79\, F$

$$F_{CB} = \frac{1,79}{0,7998} F = 2,238\, F$$

F_{CB} = 2,238 F

Reemplazar F_{AC} y F_{CB} en la ecuación 3

$F_{CD} – 0,507 F_{AC} - 0,6\, F_{CB} = 0$ ECUACION 3

$F_{CD} – 0,507\,(2,078\, F) - 0,6\,(2,238\, F) = 0$

F_{CD} – 1,053 F - 1,342 F = 0

F_{CD} = 1,053 F + 1,342 F

F_{CD} = 2,395 F

F_{AB} = 1,317 F
F_{AC} = 2,078 F
F_{CB} = 2,238 F
F_{CD} = 2,395 F
F_{DB} = 0

LA ESTRUCTURA MAS CRITICA ES F_{CD}
2,395 F = 20

$$F = \frac{20}{2{,}395} = 8{,}35\ KN$$

F = 8,35 KN